Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 1st Semester B.Sc (Hons.) Final Examination-2022
Department of Mathematics
MAT 1101: Fundamentals of Mathematics (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B. Answer any FIVE (5) questions from the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Given, $U = \{a, b, c, d, e, f, g\}, A = \{a, b, e, g\}, B = \{c, d, e\}, D = \{b, e, g\}$. Determine $(i)(A \cap B) \cup D'$ $(ii)(A \cap D)' \times B$ [4]

   (b) What is Tautology and Fallacy. [5]
   Construct a truth table for the statements $\sim (p \wedge q) \vee \sim (q \Leftrightarrow p)$.

   (c) For any three sets A, B and C prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. [5]

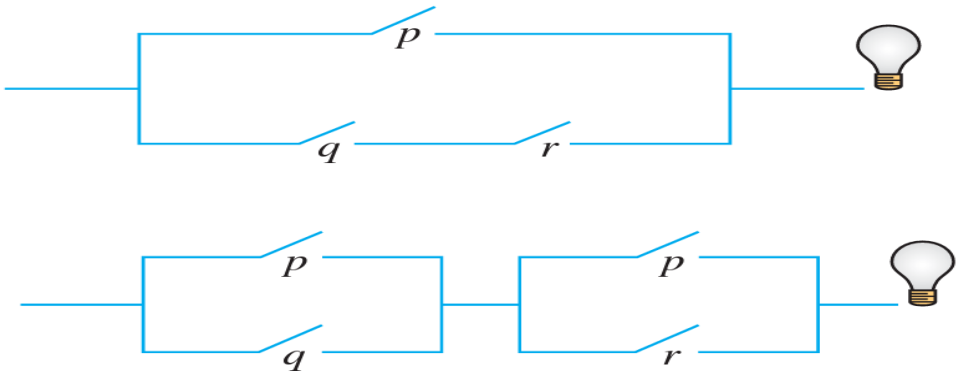
2. (a) Define Equivalence Relation with an example. Prove that the relation **"Congruence Modulo of m"** is an equivalence relation in the sets of integers. [6]

   (b) Liberty Travel surveyed **125** potential customers. The following information was obtained. **68** wished to travel to Hawaii. **53** wished to travel to Las Vegas. **47** wished to travel to Disney World. **34** wished to travel to Hawaii and Las Vegas. **26** wished to travel to Las Vegas and Disney World. **23** wished to travel to Hawaii and Disney World. **18** wished to travel to all three destinations. [8]
   Use a Venn diagram to answer the following questions. How many of those surveyed
   (i) did not wish to travel to any of these destinations?
   (ii) wished to travel only to Hawaii?
   (iii) wished to travel to Disney World and Las Vegas, but not to Hawaii?
   (iv) wished to travel to Disney World or Las Vegas, but not to Hawaii?
   (v) wished to travel to exactly one of these destinations?

3. (a) Define function and inverse function as relation. Find the domain and range of the function $f(x) = \frac{2}{\sqrt{x}}$. Also find f^{-1}, and hence find the domain and range of f^{-1}. [4]

   (b) State De Morgan's laws and prove them by constructing truth tables. [5]

   (c) Determine whether the two circuits are equivalent or not. [5]



4. (a) State and proved De Moivre's theorem. [7]

   (b) Find the modulus and argument of $-1 - i$. [7]
   Describe the equation $|z + 2i| + |z - 2i| = 6$, where $z = x + iy$.

5. (a) Find the sum of the following series. [4+5]

$$(i)\ 1 + (1 + 4) + (1 + 4 + 9) + (1 + 4 + 9 + 16) + \cdots \text{ to } n \text{ terms.}$$

$$(ii)\ \frac{1}{2 \cdot 4} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} + \cdots\cdots$$

   (b) Show that $\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} - \cdots \infty = \ln\left(\frac{4}{e}\right)$. [5]

6. (a) Prove that for any number of unequal positive quantities is [7]

$$A.M > G.M > H.M$$

   (b) Write Cauchy-Schawartz inequality. If x, y, z are positive real numbers such as [4]
   $x^2 + y^2 + z^2 = 27$, then applying Cauchy-Schawartz inequality shows that
   $x^3 + y^3 + z^3 \geq 81$.

   (c) Prove that $(n!)^2 > n^n$. [4]

7. (a) If a, b, c are the roots of the equation $x^3 + qx + r = 0$ form the equation whose [4]
   roots are $(b - c)^2, (c - a)^2, (a - b)^2$.

   (b) Find the sum of the fourth powers and sixth powers of the roots of the equation [5]
   $x^4 - x^3 - 7x^2 + x + 6 = 0$.

   (c) Find the nature of the roots of the equation $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$. And [5]
   then solve the equation whose roots are in Arithmetic progression.

8. (a) Four candidates A, B, C, and D are running for class president and receive the num- [8]
   ber of votes shown in the table.

6	7	3	4
D	C	A	B
C	B	D	A
B	D	B	C
A	A	C	D

   (i) How many votes were cast in the election?
   (ii) How many first-place votes are needed for a majority?
   (iii) Did any candidate receive a majority of first place votes?
   (iv) Who is the winner using the plurality method?

   (b) Determine the standard quotas for the clinics B, C, D and E of the First Physicians [6]
   Organization and complete the following table.

Clinic	A	B	C	D	E	Total
Patients	246	201	196	211	226	1080
Standard quota	13.67					

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 1st Semester B.Sc (Hons.) Final Examination-2022
Department of Mathematics
MAT 1103: Differential Calculus (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B. Answer any FIVE (5) questions from the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define the domain and range of a function. Write down the domain and range of the following function(any two). [2+4]

$$(i)\ y = \frac{x-4}{x^2-16} \qquad (ii)\ y = \ln\left(\frac{1+x}{1-x}\right) \qquad (iii)\ y = \sqrt{4-x^2}$$

(b) Test the symmetry of $(i) y = \frac{4x^2}{x^2+1}$ and $(ii) y = 3x^5 - 5x^3$. Also determine which function is even, odd or not. [4]

(c) Sketch the following function (any two). [4]

$$(i)\ y = 5 + \frac{1}{x-2} \quad (ii)\ y = (x+2)(x-3)^2 \quad (iii)\ y^2 = 4 - x$$

2. (a) What is limit? What is the difference between $f(a)$ and $\lim_{x\to a} f(x)$. [2]

(b) Find the following limit. [2]

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$$

(c) Given that $\lim_{x\to a} f(x) = 2, \lim_{x\to a} g(x) = -4$. Find the limit of [4]

$$(i) \lim_{x\to a} \sqrt[3]{6+f(x)} \qquad (ii) \lim_{x\to a} \{f(x) + 2g(x)\}$$

(d) Define continuity of a function $f(x)$ at $x = a$. Given a function [2+4]

$$f(x) = \begin{cases} cx - 2, & \text{if } x \le -1 \\ 4x + 5, & \text{if } -1 < x < 2 \\ -dx + 17, & \text{if } x \ge 2 \end{cases}$$

Find the values of c and d so that the function $f(x)$ is continuous for all real x.

3. (a) Write the conditions for the differentiability of $f(x)$ at $x = a$. [7]
If $f(x) = |x+2| + |x-3|$, then sketch the curve and check the differentiability at $x = 3$.

(b) Discuss the continuity and differentiability of the function $f(x)$ at $x = \frac{\pi}{2}$, [7]

$$\text{where} \quad f(x) = \begin{cases} 1, & \text{if } x < 0 \\ 1 + \sin x, & \text{if } 0 \le x < \frac{\pi}{2} \\ 2 + \left(x - \frac{\pi}{2}\right)^2, & \text{if } x \ge \frac{\pi}{2} \end{cases}$$

4. (a) Show that $y = e^{-2x}$ satisfies the equation $\frac{d^2y}{dx^2} - 4y = 0$. [2]

(b) If $F(x) = f(x\,g(x))$ then find the value of $F'(1)$ where $f'(1) = 1, g(1) = 1, g'(1) = 1$. [3]

(c) Find the differential coefficient $\frac{dy}{dx}$ of the following functions (any three). [3*3=9]

$(i)\ \sin(x^2y^2) = x$ $\quad$ $(ii)\ y = \left(2^x + x^2\right)^{f(x)}$

$(iii)\ e^{xy} = 2x + y$ $\quad$ $(iv)\ x^py^q = (x+y)^{(p+q)}$

$(v)\ y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

5. (a) If $\lim_{x\to 2} \frac{f(x)-8}{x-2} = 6$, find $\lim_{x\to 2} f(x)$. [4]

(b) Determine all horizontal and vertical asymptotes for $f(x) = \frac{1}{x^2-4}$. Justify with graph. [5]

(c) State Intermediate Value Theorem (IVT). Verify this theorem for the function $f(x) = 2x - x^2$ on the interval $[-2, -1]$. [5]

6. (a) State and prove Leibnitz's theorem. [4]

(b) If $y = \left(\sin^{-1} x\right)^2$ prove that : [6]
(i) $(1-x^2)\,y_2 - xy_1 - 2 = 0$,
(ii) $(1-x^2)\,y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$.

(c) Find local linear approximation of $f(x) = 7\sqrt{(6+x)}$ at $x = 0$ and use it to approximate the value of $7\sqrt{6.1}$ and $7\sqrt{5.9}$ [4]

7. (a) Evaluate the following limits using L'Hôpital's rule (any two). [5]

$(i)\ \lim_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$ $\quad$ $(ii)\ \lim_{x\to 0} \log_{\tan^2 x}(\tan^2 2x)$ $\quad$ $(iii)\ \lim_{x\to 0^+} \left(x^2 \ln x\right)$

(b) The rate of change of the radius of a circle is $\frac{1}{\pi}$. Find the rate of change of [4]
(i) circumferential length and
(ii) area of the circle at the instant when the radius is 2 cm.

(c) An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume? [5]

8. (a) If the percentage change side of a cube is 3.5% then find the percentage change of its volume and surface area. [5]

(b) State Taylor's theorem. Expand $f(x) = \sec x$ in power of $\left(x - \frac{\pi}{4}\right)$ with remainder in Lagrange form. [5]

(c) State Maclaurin's theorem. Find cubic expansion of $f(x) = e^x \sin x$ at $x = 0$. [4]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 1st Semester B.Sc (Hons.) Final Examination-2022
Department of Mathematics
MAT 1105: Analytic Geometry (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B. Answer any FIVE (5) questions from the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) What do you mean by analytic geometry? Convert the following in polar to Cartesian coordinates, Cartesian to polar coordinates (where applicable) and draw the graph in both coordinates by mentioning the points in detail. [5]
$(i)\ (-4, -\frac{4\pi}{3})$ $(ii)\ \left(2\sqrt{2}, \frac{3\pi}{4}\right)$ $(iii)(-1, -\sqrt{3})$

   (b) What is the locus of a point? If the equation of the base of an equilateral triangle is $x + y - 2 = 0$ and the vertex is $(2, -1)$, then find the length of the side of the triangle. [5]

   (c) The gradient of one of the straight lines of $ax^2 + 2hxy + by^2 = 0$ is twice that of the other. Show that $8h^2 = 9ab$. [4]

2. (a) Define orthogonal transformation. Find the value of a for which the transformation [7]

$$x = \frac{1}{3}x' + 6y' + a$$
$$y = \frac{1}{3}x' + 3y' + a$$

   will transform the equation $x^2 - 2xy + y^2 - 3x + 6y - 12 = 0$ into $(y')^2 = -\frac{1}{9}x'$.

   (b) Find the angle through which the axes are to be rotated so that the equation $x\sqrt{3} + y + 6 = 0$ may be reduced to the form $x = c$. Determine the value of c. [7]

3. (a) To prove that a homogeneous equation of second degree always represent a pair of straight lines through the origin. [4]

   (b) Show that the equation $3y^2 - 8xy - 3x^2 - 29x + 3y - 18 = 0$ represent a pair of straight lines. Also find their point of intersection and angle between them. [5]

   (c) If the straight lines represent by $x^2(\tan^2\theta + \cos^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ makes angle α and β with x axis, then show that $\tan\alpha - \tan\beta = 2$. [5]

4. (a) What is eccentricity of a conic? Prove that every conics section represent a second degree equation. [6]

   (b) For the equation $25x^2 + 2xy + 25y^2 - 130x - 130y + 169 = 0$. [8]
(i) Determine which conic the equation represents.
(ii) Find the rotation angle required to eliminate the xy - term ordinate system.
(iii) Transform the equation into its standard form.
(iv) Draw the graph of the resulting conic.

5. (a) A tangent to the parabola $y^2 = 8x$ makes an angle of 45^0 with the straight line $y = 3x + 5$. Find its equation and point of contact. [4]

   (b) Find the polar equation of a conics. Also, sketch the graph of $r = \dfrac{2}{1 - \cos\theta}$ in polar coordinates. [5]

   (c) What is pole and polar of a conic? If $x^2 - 3y^2 - 6xy + 5x - 2y + 3 = 0$ is a conics and $(1, -2)$ is a pole then find the equation of polar. [5]

6. (a) The line joining the points $(1, 8, -1)$ and $(4, -4, 2)$ meets the ZX and XY planes at P and Q respectively. Find the coordinates and distance between them. [4]

   (b) If P, Q, R, S are the points $(3, 4, 5), (4, 6, 3), (-1, 2, 4)$ and $(1, 0, 5)$. Find the projection of RS in PQ. [4]

   (c) If a line makes angle α, β, γ and δ with four diagonals of a cube, then prove that [6]

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$$

7. (a) Given that two lines $\dfrac{x-1}{-3} = \dfrac{y+1}{4} = \dfrac{z-4}{-2}$ and $\dfrac{x-3}{-5} = \dfrac{y-2}{1} = \dfrac{z+2}{4}$. Examine whether the lines intersect or skew lines. [4]

   (b) Find the equation of a plane passes through the point $(1, 0, -1)$ and $(2, 1, 3)$ and perpendicular to the plane $2x + y + z = 1$. [5]

   (c) Find the equation of a planes which bisect the planes $3x - 2y + 6z + 8 = 0$ and $2x - y + 2z + 3 = 0$. [5]

8. (a) (i) Find the length and equation of the shortest distance (SD) between the lines [8]

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

   (ii) Does the line intersect with other?

   (iii) Find also where the equation SD intersects the given line.

   (b) Find the equation of the paraboloid with its vertex at $(0, 0, 0)$ axis, along the z-axis and passing through the points $(2, 0, 3)$ and $(1, 2, 3)$. Also, sketch the paraboloid. [6]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 1st Semester B.Sc (Hons.) Final Examination-2022
Department of Mathematics
STA 1107: Basic Statistics (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B. Answer any FIVE (5) questions from the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Distinguish between (i) population and sample, (ii) parameter and statistic and (iii) primary and secondary data. [6]

   (b) Define frequency and frequency distribution. The following figure shows the marks obtained by some students of Chemistry department. [8]

37	42	41	46	34	35	43	19	26	48
25	33	30	34	13	22	14	39	23	38
11	18	29	25	26	25	44	38	27	18
23	27	20	21	49	15	25	27	11	45

   (i) Taking suitable class interval, construct a frequency distribution of the data. (ii) Present the frequency distribution by a suitable diagram.

2. (a) Indicate which of the following variables are quantitative and which are qualitative: (i) Number of persons in a family (ii) Colors of cars (iii) Marital status of people (iv) Time to commute from home to work [2]

   (b) The expenditure of **1000** families is given as under: [6]

Expenditure (Taka in thousand):	40-60	60-80	80-100	100-120	120-140
No. of families:	50	f_1	500	f_2	50

   The median and mean for the distribution are both $Tk.87.50$ Calculate the missing frequencies f_1 and f_2.

   (c) The frequency distribution of the number of orders received each day during the past **65** days at the office of a mail-order company. [6]

Number of Orders	10-12	13-15	16-18	19-21	22-24	25-27
Number of Days	4	12	20	14	10	5

   Calculate Quartile deviation, D_6, and P_{85}.

3. (a) Difference between exclusive and inclusive interval with example. [3]

   (b) The first four moments of a distribution about the value **4** are $-1.5, 17, -30$ and **120**. Comment on the skewness and kurtosis of the distribution. [4]

   (c) You are given below the daily wages paid to the workers in the two factories X and Y: [7]

Daily Wages	12-14	14-16	16-18	18-20	20-22	22-24	24-26
Factory X	15	30	44	60	40	14	7
Factory Y	25	40	60	35	12	15	5

   Which factory shows greater uniformity in its wage structure? Explain.

4. (a) What is Bi-variate data? [2]

   (b) Two variables x and y have the following Bi-variate distribution. [12]

	y values				
x values	7	10	13	16	19
1.5	3	2	3	7	2
3.5	3	2	3	3	1
5.5	-	-	1	-	-
7.5	1	3	1	-	1
9.5	-	-	-	1	-

Calculate the correlation ratio of y on x and that of x on y.

5. (a) What is regression analysis? Write the primary objective of a regression analysis to build a simple regression equation. [4]

   (b) The correlation coefficient for the data x_1, x_2, x_3 are $r_{13} = r_{23} = 0.778$ and $r_{12} = 0.445$. Compute the multiple correlation coefficient $R_{2.13}$ and partial correlation coefficient $r_{23.1}$ [4]

   (c) A factory produces two types of lamps. In an experiment on the working life of these lamps in the following: [6]

Lengths of life (Hours)	10-12	12-14	14-16	16-18	18-20	20-22	22-24
Type A	13	33	42	62	40	10	8
Type B	23	43	60	40	15	15	7

(i)Draw the ogive curves for both types of lamps on a single diagram.
(ii) Comment on the diagram for any unusual features.

6. (a) What do you mean by correlation? Show that the Pearson's correlation coefficient is independent of origin and scale of measurement. [7]

   (b) During the term, 10 pupils obtained the following scores in mathematics and science (out of 50). [7]

Math	29	38	22	27	32	41	20	32	36	31
Science	27	34	24	30	29	42	21	33	34	29

(i) Draw a scatter plot to represent the data. Can you conclude any possible correlation from the plot?
(ii) Compute Pearson's correlation coefficient and comment on your result.

7. (a) A nutrition study recorded a sample of **7** children under **5** years of age-weighted and their monthly family incomes in '**000** taka. The result is shown in the accompanying table. [7]

Family income (in '**000** taka)	13	20	34	24	16	30	36
Weight in kg	15	19	21	16	12	16	18
Estimated weight	15.2	16.4	19.0	17.2	15.7	18.2	19.3

Compute SST, SSR, SSE, r^2, and Standard Error.

(b) Seven students appearing at an examination were evaluated by two independent examiners out of 100 marks. Table below shows these marks. [7]

Examiner	Marks assigned						
Examiner 1	60	65	70	62	75	80	65
Examiner 2	55	60	74	65	70	75	60

(i) Rank the data and hence compute the rank correlation coefficient.
(ii)Using original data, Calculate the simple correlation coefficient.
(iii) Comment on both results.

8. (a) What do you mean by dispersion of a variable? Prove that the variance is minimized if it computed from the arithmetic mean. [7]

(b) What is coefficient of variation? What are its advantages over the other measures of dispersion? [3]

(c) the average weekly wage in a factory had increased from Tk. 8000 to Tk. 12000 as a result of negotiation between the workers and officers. Alongside, the standard deviation had decreased from Tk.150 to Tk. 100. Can we conclude that after negotiation, the wage has become higher and uniform? [4]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 1st Semester B.Sc (Hons.) Final Examination-2022

Department of Mathematics

PHY 1109: Mechanics, Waves, and Properties of Matters (3 Credits)

Time: 3 Hours **Full Marks: 70**

N.B: Figures shown in the right margin indicate full marks.
Answer any 5 out of 8 questions.

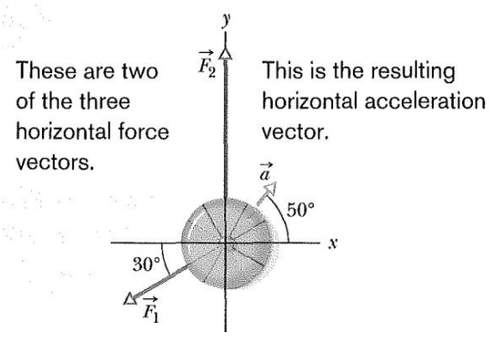
1 **(a)** Define torque and moment of inertia. **3**

(b) Show that the trajectory of a projectile is parabolic. Hence find out the maximum horizontal range of that projectile. **7**

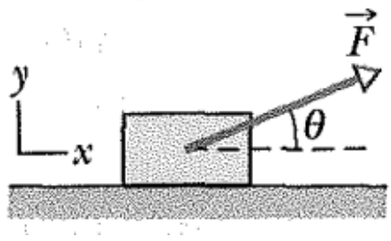
(c) A soccer player kicks a ball at an angle of 36° from the horizontal with an initial speed of 15.5 m/s. Assuming that the ball moves in a vertical plane, find (i) the time at which the ball reaches the highest point of its trajectory and (ii) its velocity when it strikes the ground. **4**

2 **(a)** State and explain Newton's three laws of motion. **4**

(b) In the overhead view of figure, a 2.0 kg cookie tin is accelerated at 3.0 m/s^2 in the direction shown by $\vec{a}$, over a frictionless horizontal forces, only two of which are shown: $\vec{F}_1$of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N. What is the third force $\vec{F}_3$ in unit-vector notation and in magnitude-angle notation? **5**



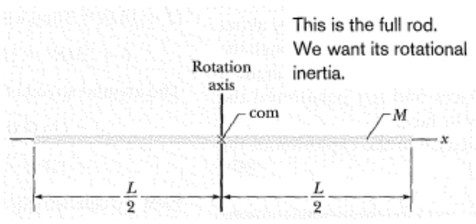
(c) As in the figure below, a block of mass m=3.0 kg slides along a floor while a force $\vec{F}$ of magnitude 12.0 N is applied to it at an upward angle θ. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.40$. We can vary θ from 0 to 90° (the block remains on the floor). What θ gives the maximum value of the block's acceleration magnitude a? **5**



3. **(a)** Write down the equation of a plane progressive wave. Explain each term in the equation. **2**

(b) Show that for rotational motion $\tau = I\alpha$, where the symbols have their usual meanings. **4**

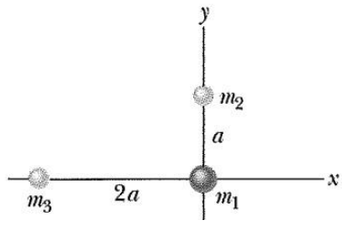
(c) Figure shows a thin uniform rod of mass M and length L, on an x axis with the origin at the rod's center. (i) Calculate the moment of inertia of the rod about the perpendicular axis through the center. (ii) By using the parallel axis theorem, calculate the moment of inertia of the rod about a rotational axis perpendicular to the rod and through the left end. **8**



4. **(a)** Define escape speed. Find out an expression for the escape speed of an object from the earth's surface. **1+4**

(b) Suppose an object of mass m is released in a tunnel through the center of the earth from the north pole to the south. Show that the motion of the object will be oscillatory. **4**

(c) The figure shows an arrangement of three particles, particle 1 of mass m_1=6.0 kg and particle 2 and 3 of mass m_2=m_3=4.0 kg, and distance a=2.0 cm. What if the net gravitational force $\vec{F}_{1,net}$ of particle 1 due to the other particles? **5**



5 **(a)** Describe simple harmonic motion and damped harmonic motion. **3**
(b) Derive the differential equation of a damped and forced harmonic motions. **7**
(c) In a damped oscillator, given that the mass of a block $m = 250$ g, spring constant $k =$ 85 N/m, damping constant $b = 0.07$ kg/sec. In how many periods of oscillation will the mechanical energy of the oscillator drop to one-half of its initial value? **4**

6. **(a)** Show that the speed of sound in a medium can be given by $v = \sqrt{\frac{B}{\rho}}$, where B is the bulk modulus and ρ is the density of the medium. **5**
(b) If an earplug decreases the sound level of the sound waves by 20 dB, what is the ratio of the final intensity I_f (after you wear the earplug) to the initial intensity I_i (before you wear the earplug)? **4**
(c) Derive the general equation for Doppler effect of sound. **5**

7. **(a)** Define Poisson's ratio. Show that $-1 \leq \sigma \leq 0.5$ **1+6**
(b) What is a cantilever? Show that if a cantilever is loaded with weight W at the free end and the weight of the cantilever is ineffective, the free end is depressed by $\frac{WL^3}{3YI_g}$, where the symbols have their usual meanings. **7**

8. **(a)** Distinguish between streamline and turbulent flow of a liquid. **3**
(b) Derive Bernoulli's equation for a fluid in streamline motion. **8**
(c) A plate of metal 10^{-2} m^2 area rests on a layer of castor oil 2×10^{-3} m thick whose coefficient of viscosity is 1055 Ns/m^2. Calculate the horizontal force required to move the plate with a uniform speed of 3×10^{-2} m/s. **3**