Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1^{st} Year 2^{nd} Semester B.Sc (Hons.) Final Examination-2022
Department of Mathematics
MAT 1201: Linear Algebra I (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) What is idempotent matrix? Prove that $\begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix}$ is an idempotent matrix. [5]

   (b) Prove that every square matrix can be express as a sum of symmetric and skew-symmetric matrix. [4]

   (c) If $\boldsymbol{A} = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$ and $(\boldsymbol{A}+\boldsymbol{B})^2 = \boldsymbol{A}^2 + \boldsymbol{B}^2$, find the value of $\boldsymbol{a}$ and $\boldsymbol{b}$. [5]

2. (a) For which values of $\boldsymbol{k}$ will the following system of equations has exactly one solution, more than one solution, and no solution? [6]

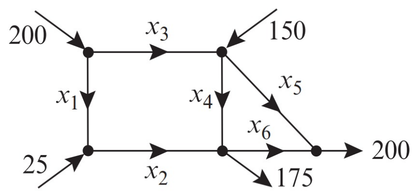
$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + \left(k^2 - 14\right) z &= k + 2. \end{aligned}$$

   (b) The accompanying figure shows known flow rates of hydrocarbons into and out of a network of pipes at an oil refinery.

   i. Set up a linear system whose solution provides the unknown flow rates. [2]

   ii. Solve the system for the unknown flow rates. [4]

   iii. Find the flaw rates and directions if $\boldsymbol{x_4} = \mathbf{50}$ and $\boldsymbol{x_6} = \mathbf{0}$. [2]



3. (a) Define the terms: Euclidean inner product, norm, and distance in $\mathbb{R}^n$. Let $\underline{u} = (4, 1, 2, 3)$, $\underline{v} = (0, 3, 8, -2)$, and $\underline{w} = (3, 1, 2, 3)$ be vectors in $\mathbb{R}^4$. Evaluate each expression [6]

$$(i)\ \|3\underline{u} - 5\underline{v} + \underline{w}\|, \qquad (ii)\ \left\| \frac{1}{\|\underline{w}\|}\underline{w} \right\|.$$

   If $\|\underline{u} + \underline{w}\| = 1$ and $\|\underline{u} - \underline{w}\| = 5$ then compute $\underline{u} \cdot \underline{w}$.

   (b) Define symmetric and skew-symmetric matrices with examples. Take into account $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 0 \end{pmatrix}$ and $\boldsymbol{B} = \boldsymbol{B}^T$. Is the matrix $(\boldsymbol{A}+\boldsymbol{B})^3$ symmetric? [4]

   (c) Consider the matrices $\boldsymbol{A} = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}$, $\boldsymbol{C} = \begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}$. [4]
   Compute Trace $\left(\boldsymbol{B}^T\boldsymbol{A}^T + 2\boldsymbol{C}^T\right)$ (where possible).

4. (a) Define a vector space. Consider the set $V = \left\{ \begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix} \in M_2(\mathbb{R}) | a, b \in \mathbb{R} \right\}$. Test whether V is a vector space or not. If not, then list all axioms that fail to hold. [4]

(b) Define a subspace of a vector space with an example. Does the subset $W = \{(a, b, c, d) \in \mathbb{R}^4 | a \geq b\}$ becomes a subspace of $\mathbb{R}^4$? [4]

(c) What is meant by a linear combination? If $S = \{\underline{v}_1, \underline{v}_2, \cdots, \underline{v}_n\}$ is a set of vectors in a vector space V, then prove that the set $W = L(S)$ of all linear combinations of vectors in S is a subspace of V. [6]

Express the polynomial $\underline{p}(x) = 7 + 8x + 9x^2$ as a linear combination of the polynomials $\underline{p}_1(x) = 2 + x + 4x^2$, $\underline{p}_2(x) = 1 - x + 3x^2$ and $\underline{p}_3(x) = 3 + 2x + 5x^2$.

5. (a) Define direct sum of vector space. Take into account $U = \{(a, b, c) \in \mathbb{R}^3 : a = b = c\}$ and $W = \{(0, b, c) \in \mathbb{R}^3 : b, c \in \mathbb{R}\}$ be two subspaces of $\mathbb{R}^3$. Show that $\mathbb{R}^3 = U \bigoplus W$. [3]

(b) How do we define basis and dimension of a vector space? Determine a basis for and the dimension of the solution space of the homogeneous system [5]

$$\begin{aligned} 2x_1 + 2x_2 - x_3 + x_5 &= 0 \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0 \\ x_1 + x_2 - 2x_3 - x_5 &= 0 \\ x_3 + x_4 + x_5 &= 0 \end{aligned}$$

(c) Let U and W be the subspaces of $\mathbb{R}^4$ defined by [6]

$$U = \{\underline{a} = (a_1, a_2, a_3, a_4) \in \mathbb{R}^4 \mid a_2 - 2a_3 + a_4 = 0\}$$
$$W = \{\underline{a} = (a_1, a_2, a_3, a_4) \in \mathbb{R}^4 \mid a_1 = a_4,\ a_2 = 2a_3\}$$

Find a basis and the dimension of U, W and $U \cap W$, respectively.

6. (a) Let us consider the bases $B = \{\underline{u}_1, \underline{u}_2\}$ and $B' = \{\underline{u}'_1, \underline{u}'_2\}$ where [8]

$$\underline{u}_1 = (1, 0), \underline{u}_2 = (0, 1), \underline{u}'_1 = (1, 1), \underline{u}'_2 = (2, 1)$$

(i) Find the transition matrix P from B' to B.
(ii) Find the transition matrix Q from B to B'.
(iii) Prove that $PQ = I$.

(b) Find the coordinate vector of $\underline{v}$ related to the basis $S = \{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ for $\mathbb{R}^3$ where $\underline{v} = (2, -1, 3)$, $\underline{v}_1 = (1, 0, 0)$, $\underline{v}_2 = (2, 2, 0)$, and $\underline{v}_3 = (3, 3, 3)$. [6]

7. (a) Define Kernel and Image of a linear transformation. If $T : U(F) \to V(F)$ is a linear transformation then prove that $\text{Im}(T)$ is subspace of U and $\ker(T)$ is subspace of V. [7]

(b) Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - 2y + z, x + 5y + 3z, 3x + y + 5z)$. Find the rank and nullity of the linear transformation of T. [7]

8. (a) What is eigenvalues and eigenvectors of a linear operator? [2]

(b) If $\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_n$ are eigenvectors of a square matrix A corresponding to the distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ respectively then the eigenvectors are linearly independent. [5]

(c) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$. Also find a matrix P such that $P^{-1}AP$ is a diagonal matrix. [7]

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1st Year 2nd Semester B.Sc (Hons.) Final Examination-2022
Department of Mathematics
MAT 1202: MATHEMATICA Lab (3 Credits)

Time: 03.00 Hours **Full Marks: 60**

Answer all of the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Input a number $\boldsymbol{n \geq 20}$ from the keyboard and check the number prime and perfect or not. [7]

   (b) Let us consider a sequence [8]

$$\mathbf{0, 1, 2, 3, 6, 11, 20, 37, 68, \ldots\ldots}$$

   Now print all the numbers of the sequence up to $\mathbf{100}^{\text{th}}$ term and find the sum of all digits of $\mathbf{100}^{\text{th}}$ term of the sequence.

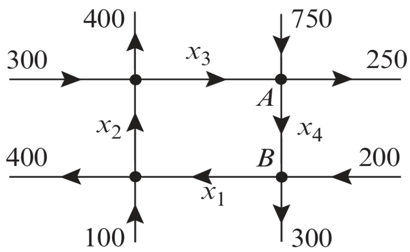
2. (a) Find the value of $\boldsymbol{\lambda}$ for which the ellipse $\boldsymbol{9x^2 + \lambda y^2 = 9\lambda}$ passes through the point $\mathbf{(6, 2)}$. Hence,

   i. Sketch the ellipse and locate the foci and vertices in the same figure with blue heavy dots. Also, find its eccentricity. [6]

   ii. Draw the integer points inside the ellipse with heavy dots. How many integer points are there inside the ellipse? [4]

   (b) Determine the equation of the curve $\boldsymbol{x^2 - y^2 - 2\sqrt{2}x - 10\sqrt{2}y + 2 = 0}$ when, the axes are inclined at $\mathbf{45^\circ}$ to the original axes. [5]

3. (a) The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour. Solve the system for the unknown flow rates. [7]



   (b) Consider the matrix $\boldsymbol{A} = \begin{bmatrix} \mathbf{7} & \mathbf{2} & \mathbf{2} \\ \mathbf{-6} & \mathbf{-1} & \mathbf{2} \\ \mathbf{6} & \mathbf{2} & \mathbf{-1} \end{bmatrix}$. [8]

   i. Find the eigenvalues of $\boldsymbol{A}$.

   ii. Verify Cayley-Hamilton theorem for $\boldsymbol{A}$.

   iii. Find the matrix $\boldsymbol{P}$ that diagonalizes $\boldsymbol{A}$.

4. (a) Evaluate the integral using **mid point** Riemannian Sum. [7]

$$\text{i.} \int_{-1}^{2} (x+2)\,dx, \quad \text{ii.} \int_{0}^{1} \sqrt{1-x^2}\,dx$$

   (b) Sketch the graphs of the following functions with horizontal and vertical asymptote in different colors: [8]

$$y = \frac{x^2}{x^2 - x - 2},$$

   Also, find the domain and range, and confirm that your obtained results are consistent with the graphs.

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1^{st} Year 2^{nd} Semester B.Sc (Hons.) Final Examination-2022

Department of Mathematics

MAT 1203: Integral Calculus (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B. Answer any FIVE (5) questions from the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define the antiderivative of a function. Use the antiderivative method to find the area under the curve $y = x^2$ over the interval $[0, 2]$. [4]

   (b) Find the midpoint approximations of the area under the curve $y = 9 - x^2$ over the interval $[0, 3]$ with $n = 10$, and $n = 15$. [5]

   (c) $\int_0^3 |x - 4| dx$. Sketch the region whose signed area is represented by the definite integral, and evaluate the integral using an appropriate formula from geometry, where needed. [5]

2. (a) Evaluate the following indefinite integrals [6]

$$(i) \int \frac{dx}{a\cos x + b\sin x} \qquad (ii) \int \frac{dx}{\sqrt{(x-\alpha)(\beta - x)}}$$

   (b) Evaluate the following definite integrals [8]

$$(i) \int_0^{\pi/2} \frac{d\theta}{1 + 2\cos\theta} \qquad (ii) \int_0^{\pi/2} \ln \sin x \; dx$$

.

3. (a) State Walli's formula. If $U_n = \int_0^1 x^n \arctan x \, dx,$ then prove that [7]

$$(n+1)U_n + (n-1)U_{n-2} = \frac{\pi}{2} - \frac{1}{n}.$$

   (b) Using the fundamental theorem of calculus or otherwise evaluate: [7]

$$(i) \frac{d}{dx}\left(\int_a^x \ln\left(\sin^2 t + 1\right) dt\right) \qquad (ii) \int_a^x \frac{d}{dt}\left(\sqrt{1+t^2}\right) dt.$$

4. (a) Find the average value of the function $f(x) = \sqrt{x}$ over the interval $[1, 4]$, and find all points in the interval at which the value of f is the same as the average. [4]

   (b) Derive a reduction formula for $\int \cos^n x dx$. And hence evaluate $\int \cos^4 x dx$. [5]

   (c) Sketch the curve $f(x) = \frac{x^2}{2}$. Then find the arc length of the curve from $x = 0$ to $x = 1$. [5]

5. (a) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$. [4]

   (b) By using integration, derive the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a. [5]

(c) Intelligence Quotient (IQ) scores are distributed normally with mean **100** and standard deviation **15** . [5]
(i) What percentage of the population has an IQ score between **85** and **115** ?
(ii) What percentage of the population has an IQ above **140** ?

6. (a) Find the volume of the solid generated when the region under the curve $y = x^2$ over the interval $[0, 2]$ is rotated about the line $y = -1$. [4]

(b) Use cylindrical shells to derive a formula for the volume of the solid generated when the region is revolved about the y-axis. [5]

(c) Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis. [5]

7. (a) Define beta and gamma function. Establish the relationship between them. [5]

(b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ and $\Gamma(n+1) = n!$. [4]

(c) Prove that [5]

$$\int_0^{\pi/2} \sin^m x \ \cos^n x \ dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\,\Gamma\left(\frac{m+n+2}{2}\right)}$$

8. (a) Sketch the following polar curve (any three) [3]
(i) $r = \theta \quad (\theta \geq 0)$ (ii) $r = \sin 3\theta$ (iii) $r = a(1 - \cos\theta)$ (iv) $r^2 = 4\cos 2\theta$

(b) Find the circumference of a circle of radius a from the parametric equations [3]

$$x = a\cos t, \quad y = a\sin t \quad (0 \leq t \leq 2\pi)$$

(c) Find the entire area within the cardioid of $r = 1 - \cos\theta$. [3]

(d) Find the surface area of $r = 4 + 4\sin\theta$ when the segment from $\theta = -\frac{\pi}{2}$ to $\frac{\pi}{2}$ is revolved around the line $\theta = \frac{\pi}{2}$. [5]

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1st Year 2nd Semester B. Sc. (Hons.) Final Examination-2022

Department of Mathematics

STA 1205: Introduction to Probability (3 Credits)

Time: 3 Hours **Full Marks: 70**

Figures shown in the right margin indicate full marks. Answer any 05 out of 08 questions.

1. a. Define the following terms with example- **4**
i) Random experiment ii) Sample space iii) Event, and iv) Venn diagram

b. What are the various approaches of defining probability of an event? Make a critical review of these definitions indicating their relative merits and demerits. **4**

c. Consider an experiment of tossing two unfair coins with $P(x)=\frac{1}{9};\ x=4,5,6\ P(H)=\frac{2}{3}$ and $P(T)=\frac{1}{3}$. Find the probability that (i) exactly one head is observed; (ii) at least one head is observed; (iii) at most one head is observed. **6**

2. a. Briefly discuss classical and relative frequency approach of defining probability with relative merits and demerits. **4**

b. If $P(A) = 0.8$, $P(B) = 0.5$, and $P(A \cap B) = 0.4$, determine the following probabilities: $P(A \cup B)$, $P(A^c \cap B)$ and $P(A^c \cup B)$. **4**

c. A lot contain 15 bulbs, of which 2 are defective. An investigator selects 5 bulbs at random from the lot. Find the probability that (i) none are defective (ii) exactly two are defective (iii) at most one are defective (iv) at least one are defective. **6**

3. a. Define conditional probability. What do you mean by independence of two events? **4**

b. The following table shows the number of adults (in thousands) in the United States who were employed and unemployed in 2018 along with their gender. **4**

	Male	Female
Unemployed	1680	1580
Employed	65032	64755

Suppose that an individual will be selected from the population. Determine whether the events "the individual is unemployed" and "the individual is male" are independent. Also find the conditional probabilities of unemployed individuals.

c. State and prove Bayes' Theorem. A company produces machine components which pass through an automatic testing machine. If a component is identified as defective by the system it then be rejected. A total of 4% of the components entering the testing machine are defective. However, the automatic testing machine is not entirely reliable. In this process, 10% of the defective components will not be rejected but 5% of the non-defective components will be rejected. Suppose a randomly selected machine component passed through the testing machine, find the probability that the component is rejected by the process. Also suppose that a component is rejected by the process, find the probability that it is non-defective. **6**

4. a. Define joint probability distribution for discrete random variable. A coin is tossed 3 times. If X denotes the number of heads and y denotes the number of tails in the last two tosses, find the joint probability distribution of X and Y. **4**

b. The joint probability density of X and Y is given by **4**

$$f(x,y) = \begin{cases} \frac{x+y}{8}, & 0 \le x \le 2, 0 \le y \le 2 \\ 0, & Otherwise \end{cases}$$

i) verify that it is a joint density function
ii) Find the marginal density function of X.

c. Suppose that two continuous random variables X and Y have a joint probability density function- **6**

$$f(x,y) = \begin{cases} A(x-3), & -2 \le x \le 3, 4 \le y \le 6 \\ 0, & Otherwise \end{cases}$$

i) What is the value of A?
ii) What is $P(0 \le X \le 1 \text{ and } 4 \le y \le 5)$?
iii) Are the random variables X and Y independent?
iv) What are the expectations and variances of the random variables X and Y?
v) What is the covariance of X and Y?
vi) What is the correlation between X and Y?

5. a. Suppose that X is a continuous random variable with pdf $f(x) = 2x,\ 0 < x < 1$, find the pdf of $Y = 3X + 1$. Also find (i) $P(Y < 3)$ (ii) $P(Y > 2)$. **4**

b. Suppose that the joint pmf of X and Y is as follows **4**

$$f(x,y) = \frac{x+y}{30},\ x = 0,1,\ 2,\ y = 0,1\ 2,3.$$

Find the marginal distributions of X and conditional distribution of Y given $X = x$.

c. Suppose that X and Y have joint pdf as follows **6**

$$f(x,y) = \frac{x+y}{8},\ 0 < x < 2,\ 0 < y < 2\ .$$

(i) Find the marginal distribution of X and Y
(ii) Are X and Y independent? Verify.
(iii) Find $P(X < Y)$
(iv) Compute $P(Y < 1|X = 1)$

6. a. Define expected value of a random variable. If X and Y are two independent random variable, then prove that $E(XY) = E(X)E(Y)$. **4**

b. What is moment generating function (MGF)? Why it is so called? Explain. **4**

c. Let $X_1,\ X_2,\ \cdots,\ X_n$ be independent random variables. **6**

(i) Prove that the MGF of $Y = \sum_{i=1}^{n} X_i$ is $M_Y(t) = \prod_{i=1}^{n} M_{X_i}(t)$.
(ii) If each X_i has the pmf $f(x_i) = p^{x_i}(1-p)^{1-x_i}$ where $x_i = 0,\ 1,\ i = 1,\ 2,\ \cdots,\ n,\ 0 < p < 1$. Find the MGF of $Y = \sum_{i=1}^{n} X_i$.

7. a. Define binomial distribution. Derive moment generating function of binomial distribution. Hence find mean and variance. **4**

b. Suppose a milk factory has 20 containers and there is a probability of 0.261 that a milk container is underweight. **4**

i) What is the distribution of the number of underweight containers in a box?
ii) Calculate expected number of underweight cartons in a box and also calculate its variance.
iii) Calculate the probability that a box contains no more than three underweight containers.
iv) Calculate the probability that a box contain at least two underweight containers.

c. Write down the difference between binomial distribution and Poisson distribution. Suppose that the number of errors in a piece of software has a parameter $\lambda = 3$. This parameter immediately implies that the expected number of errors is three and that the variance in the number of errors is also equal to three. **6**

i) What is distribution of the number of errors in a piece of software?
ii) Calculate the probability that a piece of software has no errors.
iii) Calculate the probability that there are three or more errors in a piece of software.

8. a. Define normal distribution. If $X \sim N(\mu, \sigma^2)$, then prove that $f(x; \mu, \sigma^2)$ is a pdf. **4**

b. The weight of a sophisticated running shoe is normally distributed with a mean of 15 ounces and a standard deviation of 0.55 ounce. What percentage of the shoes having weight (i) at most 16 ounces? (ii) more than 16.5 ounces? (iii) between 14.5 ounces and 15.5 ounces? **4**

c. Find the MGF of a normal random variable and hence find its cumulants generating function, mean and variance. **6**

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1st Year 2nd Semester B. Sc. (Hons.) Final Examination-2022

Department of Mathematics

PHY 1207: Electricity and Magnetism (3 Credits)

Time: 3 Hours **Full Marks: 70**

Figures shown in the right margin indicate full marks.
Answer any 05 out of 08 questions.

1. **a.** State and explain coulomb's law and define 1 C of charge. **[4]**

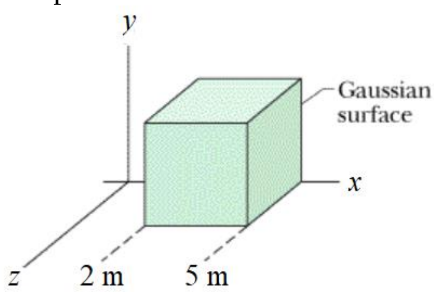
b. Derive an expression for the electric potential due to a charged particle with necessary diagram. **[6]**

c. A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude 6.2×10^{-30} C.m. (i) How far apart are the molecule's centers of positive and negative charge? (ii) If the molecule is placed in an electric field of 2×10^{5} N/C, what maximum torque can the field exert on it? **[4]**

2. **a.** Explain electric flux for a flat surface with necessary diagram and mathematical expression. **[4]**

b. Apply Gauss' law to derive mathematical expression of the electric field for charge distributions which have cylindrical symmetry. **[4]**

c. A non-uniform electric field given by $\vec{E} = 4x\hat{\text{i}} + 5\hat{\text{j}}$ pierces the Gaussian surface shown below. If E is in N/C and x is in m, find the electric flux through the right surface, left surface and the top surface. **[6]**



3. **a.** Show that the capacitance of an isolated conductor of radius R is $4\pi\epsilon_0 R$. **[4]**

b. While a parallel plate capacitor remains connected to a battery, a dielectric slab is slipped between the plates. Explain quantitatively what happens to the charge, the capacitance, the potential difference, the electric field, and the stored energy. **[4]**

c. In a charging RC circuit, find the resistor's current and potential difference as functions of time. **[6]**

4. **a.** What is dielectric constant? Describe the effect on the capacitance of a capacitor if a dielectric slab is inserted between the plates of a capacitor. **[4]**

b. Derive mathematical expression for Gauss' law with dielectric and define electric displacement. **[6]**

c. A parallel-plate capacitor with capacitance C = 15 pF is charged by a battery to a potential difference of V = 12 V between the plates. The charging battery is now disconnected and a porcelain slab with dielectric constant of k = 6.5 is slipped between the plates. Find the potential energy of (i) the capacitor before the slab is inserted and (ii) the capacitor-slab device after the slab is inserted. **[4]**

5. **a.** Define resistance, resistivity and conductivity of a conductor. **[3]**

b. What do you understand by mean free time of electrons in a conductor? Derive a mathematical expression to establish the relationship between resistivity and mean free time for a conductor. **[6]**

c. What is current density? The current density J in a cylindrical wire of radius R = 3 mm is uniform across the cross section of the wire and J = 300000 A/m^2. Find the current through the outer portion of the wire between radial distance R/3 and R. **[5]**

6. **a.** State and explain Faraday's law of electromagnetic induction. What is Lenz's law? **[4]**

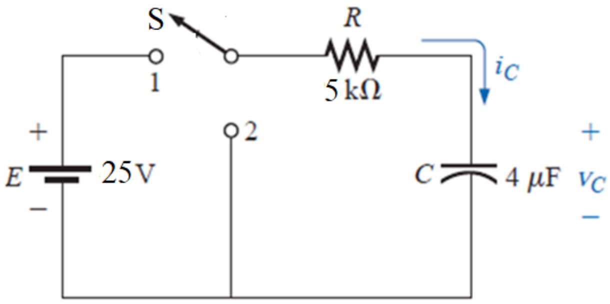
b. Show that the energy required to build up a current i in a circuit having self-inductance L is $\frac{1}{2}(Li^2)$. **[4]**

c. State and explain Gauss' law for magnetism. Differentiate between diamagnetism and paramagnetism. **[6]**

7. **a.** What is capacitive time constant? What happens to the charge on the plates of an uncharged capacitor after the first time constant of its charging phase? **[2]**

b. An uncharged capacitor of capacitance C in a series RC circuit is charged by an ideal battery of emf 'E'. Derive mathematical expression for the variation of charge 'q' on the capacitor plates, current 'i' in the circuit and potential difference 'v_c' across the capacitor over time in this charging phase. **[6]**

c. (i) Find the mathematical expressions for the transient behavior of i_C and v_C for the following circuit when the switch S is at position 1. Plot the curves of i_C, and v_C. **[6]**
(ii) After v_C has reached its final value of 25 V, the switch S is thrown into position 2. Find the mathematical expressions for the transient behavior of i_C and v_C. Plot the curves of i_C, and v_C.



8. **a.** State and explain Ampere's law. When Biot-Savart law is applicable? **[4]**

b. Using Ampere's law, find the magnetic field at a perpendicular distance d from a long, straight wire carrying a current i. **[4]**

c. Deduce an expression for the magnetic field of an ideal solenoid at all interior points. **[6]**