Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

Department of Mathematics

2^{nd} Year 1^{st} Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2101 Course Title: Real Analysis

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) State and interpret Archimedean property of real numbers. [3]

   (b) Prove that if a is a lower bound of a set A, and if a is an element of A then a is the infimum of A. [4]

   (c) Prove that if x and y are any two real numbers then $|x-y| \gg ||x|-|y||$. [3]

   (d) State and interpret order completeness axiom of real numbers. Discuss how order completeness axiom distinguishes between $\mathbb{R}$ and $\mathbb{Q}$. [4]

2. (a) Define: Interior, exterior, derived, open, and closed set. [5]

   (b) Prove that the union of a finite number of closed sets is a closed set. [2]

   (c) Is every infinite subset of R which is an ordered field, complete also? Justify your answer. [7]

3. (a) State Cauchy's convergence criterion for a sequence. [2]

   (b) For any two sequences $\{a_n\}$ and $\{b_n\}$ converging to a and b respectively, show that [6]

   $$\text{(i) } \lim(a_n b_n) = ab \quad \text{(ii) If } a_n \leq b_n \text{ for all } n \in \mathbb{N} \text{ then } a \leq b$$

   (c) State and prove Bolzano Weierstrass's theorem for sequence. [6]

4. (a) Show that the series $\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + ... + \frac{1}{n^p} + ...$ converges if $p > 1$ and diverges if $p \leq 1$. [7]

   (b) State and prove the Gauss Test. [5]

   (c) Is Raabe's Test stronger than D'Alembert's Ratio Test? Justify your answer with an example. [2]

5. (a) Prove that $f(x) = \frac{\sin^2 ax}{x^2}$ for $x \neq 0$ and $f(0) = 1$ is discontinuous at $x = 0$ unless $a = \pm 1$. [3]

   (b) Demonstrate that if a function f is continuous at a, then $|f|$ is also continuous at a, but not conversely. [4]

   (c) Define uniform continuity. Show that if a function f is continuous on a closed interval $[a, b]$, then it is uniformly continuous on $[a, b]$. [5]

   (d) Explain the geometrical meaning of the derivative. [2]

6. (a) Define upper and lower Darboux sums. If P' is a refinement of P containing p points more than P and $|f(x)| \leq k \;\; \forall \;\; x \in [a, b]$, then prove that $L(P, f) \leq L(P', f) \leq L(P, f) + 2pk\delta$, where $||P|| = \delta$. [4]

(b) Show by an example that every bounded function need not be R-integrable. [4]

(c) State and prove the necessary and sufficient condition for integrability. [6]

7. (a) Derive metric space with an example. Show that $(\mathbb{R}^n, d)$ is a metric space with the metric d defined by [8]

$$d(x, y) = \left[\sum_{i=1}^{n} (x_i - y_i)^2\right]^{\frac{1}{2}}$$

for all, $x = (x_1, x_2, ..., x_n), y = (y_1, y_2, ..., y_n)$. Name the metric space.

(b) Let (X, d) be a metric space and $d^*(x, y) = \min\{1, d(x, y)\}$ for all $x, y \in X$. Then show that d^* is a metric and d and d^* are equivalent. [6]

8. (a) Define the refinement and norm of a partition. Give an example explaining the idea. [4]

(b) Prove that if f_1 and f_2 are two bounded and integrable functions on $[a, b]$, then their product $f_1 f_2$ is also bounded and integrable on $[a, b]$. [5]

(c) Show that if f is bounded and integrable on $[a, b]$, then $|f|$ is also bounded and integrable on $[a, b]$. [5]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

Department of Mathematics

2nd Year 1st Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2103 Course Title: Differential Calculus of Several Variables

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define vector valued function. Show that the curve with parametric equations $x = t\cos t, y = t\sin t, z = t$ lies on the cone $z^2 = x^2 + y^2$, and use this fact to help sketch the curve. [4]

   (b) Show that [4]
$$\lim_{t\Rightarrow 1}[\boldsymbol{F}(t)\times \boldsymbol{G}(t)] = \left[\lim_{t\Rightarrow 1}\boldsymbol{F}(t)\right]\times\left[\lim_{t\Rightarrow 1}\boldsymbol{G}(t)\right];$$
   where $\boldsymbol{F}(t) = 2t\boldsymbol{i} - 5\boldsymbol{j} + t^2\boldsymbol{k}$ and $\boldsymbol{G}(t) = (1-t)\boldsymbol{i} + \dfrac{1}{t}\boldsymbol{k}$.

   (c) Define tangent vector. Determine the vector equation of tangent of a vector valued function. [6]

2. (a) Define Limit, Continuity, and Differentiability of a vector-valued function $\vec{r}(t)$. [3]

   (b) Sketch the graph and a radius vector of [4]
$$\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + 2\hat{k},\ 0 \le t \le 2\pi$$

   (c) A particle moves along a circular path in such a way that its x - and y coordinates at time t are [7]
$$x = 2\cos t, \quad y = 2\sin t$$
   (i) Find the instantaneous velocity and speed of the particle at time t.
   (ii) Sketch the path of the particle, and show the position and velocity vectors at time $t = \pi/4$ with the velocity vector drawn so that its initial point is at the tip of the position vector.
   (iii) Show that at each instant the acceleration vector is perpendicular to the velocity vector.

3. (a) Find the unit tangent, unit normal, and binormal for the space curve at $(1, 0, 1)$ [4]
$$\vec{r}(t) = t\hat{i} + \sqrt{2}\ln t\hat{j} + 1/t\hat{k}.$$

   (b) Define curvature. Establish a formula for determining the radius of curvature for the curve $f(x, y) = 0$. [4]

   (c) Determine the cord of curvature through the pole of the curve $r^n = a^n\cos n\theta$. [6]

4. (a) What is the osculating plane and osculating circle? [2]

   (b) Derive a formula for finding the curvature and center of curvature of $y = f(x)$ at (x_0, y_0). [6]

   (c) Find and sketch the circle of curvature at the point $(2, 3)$ on $\frac{x^2}{4} + \frac{y^2}{9} = 2$. [6]

5. (a) Sketch the largest region in which f is continuous: [4]
(i). $f(x,y) = \sqrt{x^2 - y}$.
(ii). $f(x,y,z) = \frac{xy+1}{x^2+y^2-1}$.

(b) Consider the function [5]

$$f(x,y) = \begin{cases} \frac{-xy}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0) \end{cases}$$

Determine whether $f(x,y)$ is continuous at $(0,0)$.

(c) Suppose that $w = x^2 + y^2 - z^2$ and [5]

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi$$

Use appropriate forms of the chain rule to find $\partial w/\partial \rho$ and $\partial w/\partial \theta$.

6. (a) A heat-seeking particle is located at the point $(2,3)$ on a flat metal plate whose temperature at a point (x,y) is [4]

$$T(x,y) = 10 - 8x^2 - 2y^2$$

Find an equation for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

(b) Find parametric equations of the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point $(1,1,2)$. [5]

(c) The length, width, and height of a rectangular box are measured with an error of at most 5%. Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box. [5]

7. (a) In each part, describe the graph of the function in an xyz-coordinate system [4]
(i) $f(x,y) = 1 - x - y/3$
(ii) $h(x,y) = 2 + \sqrt{x^2 + y^2}$

(b) Determine the dimensions of a rectangular box, open at the top, having a volume of 32 ft^3, and requiring the least amount of material for its construction [5]

(c) A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box. [5]

8. (a) Define divergence and curl of a vector function. Prove that curl grad $\mathbf{r}^n = 0$. [4]

(b) Show that $\nabla\phi$ is a vector perpendicular to the surface $\phi(x,y,z) = c$, where c is a constant. [4]

(c) Show that if a rigid body is in motion, the curl of its linear velocity at any points, gives twice its angular velocity. [6]

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2nd Year 1st Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2105 Course Title: Ordinary Differential Equations-I

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Give an example of the following: [2]
   (i) A first order nonlinear initial value problem.
   (ii) A second order linear boundary value problem.

   (b) Classify the following differential equations as linear or non-linear, and state their order and degree: [4]
   i. $x\dfrac{d^3y}{dx^3} + x^2\left(\dfrac{dy}{dx}\right)^4 + x^3y = \cos x$
   ii. $\sin x\dfrac{dy}{dx} + x^5y = (1-x^2)\dfrac{d^2y}{dx^2}$
   iii. $t^2\dfrac{d^6x}{dt^6} + \left(\dfrac{d^4x}{dt^4}\right)\left(\dfrac{d^3x}{dt^3}\right) + x = t^3.$

   (c) Determine whether the initial value problem [3]

   $$\frac{dy}{dx} = \frac{2y}{x-2}, \quad y(1) = 0$$

   has a unique solution defined on sufficiently small interval $|x-1| \leq h$ about $x_0 = 1$.

   (d) Find the differential equation of the family of parabola with vertex on $x-$axis, axis parallel to the $y-$axis and distance from the focus to vertex fixed as a. Sketch some representative members of the family. [5]

2. (a) Define homogeneous differential equation . If $M(x,y)dx + N(x,y)dy = 0$ is a homogeneous differential equation, then prove that the change of variable $y = vx$ transform into a separate equation in the variables of v and x. [5]

   (b) Solve the following differential equations. [9]
   (i) $\left(y - \sqrt{x^2+y^2}\right)dx - xdy = 0, \quad y(1) = 0$
   (ii) $\dfrac{dy}{dx} + y = xy^3.$
   (iii) $ydx + (3x - xy + 2)dy = 0$

3. (a) Define integrating factor. If $M(x,y)dx + N(x,y)dy = 0$ is not exact, state possible all forms of Integrating Factor (I.F.) that make the equation is exact. [4]

   (b) Find the integrating factor of the form x^py^q for the differential equation [5]

   $$(5x^2y + 6x^3y^2 + 4xy^2)dx + (2x^3 + 3x^4y + 3x^2y)dy = 0$$

(c) Find the orthogonal trajectories of the family of ellipse having center at the origin, a focus at the point $(c, 0)$ and semi major axis at the length $2c$.. Sketch several number of these. [5]

4. (a) When a cake is removed from an oven, its temperature is measured at $300F$. Three minutes later its temperature is $200F$. How long will it take for the cake to cool off to a room temperature of $70F$? [6]

(b) A ball weighing 3/4 lb is thrown vertically upward from a point 6 ft above the surface of the earth with an initial velocity of 20 ft/sec . As it rises it is acted upon by air resistance that is numerically equal to $1/64\ v$ (in pounds) where v is the velocity (in feet per sec). How high will the ball rise? [8]

5. (a) The roots of the auxiliary equation, corresponding to a certain 12^{th} order homogeneous linear differential equation with constant coefficients, are [4]

$$-2, 3, 4, 4, 4, 4, 2+3i, 2-3i, 2+3i, 2-3i, 2+3i, 2-3i.$$

What will be the fundamental set of solutions of the differential equation?

(b) Give a convincing demonstration that the second order differential equation $ay'' + by' + cy = 0$, where a, b, c are constants, always possesses at least one solution of the form $y_1 = e^{m_1 x}$, and then must have a second solution either of the form $y_2 = e^{m_2 x}$ or of the form $y_2 = xe^{m_1 x}$, m_1 and m_2 constants. [5]

(c) $e^x \sin 2x$ is solution of the following differential equation [5]

$$\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 13\frac{dy}{dx} + 30y = 0$$

Find the general solution.

6. (a) Solve the following differential equation. [7]

$$(D^3 + 5D^2 + 17D + 13)y = 0, \quad y(0) = 0, y'(0) = 1, y''(0) = 6$$

(b) Write down the standard form of Cauchy-Euler Equation. Find the general solution of the equation $x^3y''' - 4x^2y'' + 8xy - 8y = 4\ln x$. [7]

7. (a) Find the general solution of [8]

$$(x^2 + 2x)\frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + 2y = (x+2)^2$$

given that $y = (x+1)$ and $y = x^2$ are linearly independent solutions of the corresponding homogeneous equation.

(b) Solve the following system of differential equations by systematic elimination [6]

$$x' - 4x + y'' = t^2$$
$$x' + x + y' = 0$$

8. (a) A 4-lb weight is attached to the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby stretching the spring 6in. At time $t = 0$ the weight is then struck so as to set it into motion with an initial velocity of 2ft/sec, directed downward. [7]

(i) Determine the resulting displacement and velocity of the weight as functions of the time.

(ii) Find the amplitude, period, and frequency of the motion.

(iii) Determine the times at which the weight is 1.5 in. below its equilibrium position and moving downward.

(b) A circuit has in series an electromotive force given by $E(t) = 100\sin(200t)\ V$, a resistor of $40\ \Omega$, an instructor of $0.25\ H$, and a capacitor of $4 \times 10^{-4}\ F$. If the initial current and initial charge on the capacitor are both zero, find the charge on the capacitor at any time $t > 0$. [7]

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Department of Mathematics

2^{nd} Year 1^{st} Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2107 Course Title: Mathematical Statistics

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define parameter, statistic, random sample, and probability distribution with examples. [4]

   (b) Standard errors of statistics and their large sample approximations, explain. [5]

   (c) Let Y be the random variable with probability distribution function [5]

   $$f(y) = 2y,\ 0 \leq y \leq 1$$

   Find the PDF for $U = 3Y - 1$ and hence find $E(U)$.

2. (a) Prove that the mathematical expectation of the product of a number of independent random variables. is equal to the product of their expectations. [4]

   (b) Explain characteristic functions. Write some properties of characteristic functions. [4]

   (c) Explain conditional expectation and conditional variance. Prove that the variance of X can be regarded as consisting of two parts, the expectation of the conditional variance and the variance of the conditional expectation. [6]

3. (a) Define discrete and continuous random variable. A random variable X has the following probability distribution: [4]

x:	0	1	2	3	4	5	.6	7
$P(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

   (i) Find k,

   (ii) Evaluate $P(X < 6), P(X \geq 6)$, and $P(0 < X < 5)$,

   (iii) If $P(X \leq c) > \frac{1}{2}$, find the minimum value of c, and

   (iv) Determine the distribution function of X.

   (b) Suppose Y has the following distribution $f_Y(y) = \frac{2y}{\theta}e^{-\frac{y^2}{\theta}}, y > 0$. Find the p.d.f. for $U = Y^2$ and hence find $E(Y)$ and variance of Y. [4]

   (c) State and prove Chebychev's inequality. [6]

4. (a) State Inversion theorem. [4]
   The CF for the random variabl x is $\phi_x(t) = e^{-1/2t^2}$. Find the PDF of x.

   (b) The PDF of a random variable x is, $f(x) = \frac{1}{2}e^{-|x|}; -\infty < x < \infty$, Find the CF of x. [5]

   (c) What is a sampling distribution? Derive the PDF of Chi-square distribution with n degree of freedom. [5]

5. (a) If X_1, X_2 is a random sample from $N(0, 1)$, show that $\frac{(x_1+x_2)^2}{(x_1-x_2)^2}$ has an F-distribution with 1 and 1 degree of freedom. [4]

(b) Let X_i be a random variable distributed as $N(i, i^2), i = 1, 2, 3$. Assume that X_i' 's are independent. Given an example of a statistic that has (i) a chi-square distribution with 3 degrees of freedom (df), (ii) an F - distribution with I and 2df and (iii) a t - distribution with 2 df. [4]

(c) In one sample of 8 observations, the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102:6. Test whether this difference is significant at 5 per cent level, given that the 5 per cent point of F for $n_1 = 7$ and $n_2 = 9$ degrees of freedom is 3.29. [6]

6. (a) Define Hypothesis, level of significance, p-value, and critical value with examples. [4]

(b) Explain the procedure of testing the significance of a single mean, and single variance. [5]

(c) Samples of two types of electric light bulbs were tested for the length of life and the following data were obtained: [5]

	Type I	Type II
Sample No.	$n_1 = 8$	$n_2 = 7$
Sample Mean's	$\bar{x}_1 = 1,234$ hrs.	$\bar{x}_2 = 1,036$ hrs.
Sample S.D.'s	$s_1 = 36$ hrs.	$s_2 = 40$ hrs.

Is the difference in the means sufficient to warrant that type I is superior to type II regarding length of life?

7. (a) Let X_1 and X_2 be two independent standard normal random variables. Find the distribution of $\frac{X_2 - X_1}{\sqrt{2}}$ and $\frac{(X_1 + X_2)^2}{2}$. [4]

(b) A study shows that 100 adult males from rural areas have a mean height of 62 inches with standard deviation 2 , whereas 120 adult males from urban areas have a mean height of 60 inches with standard deviation 2.5. Test the hypothesis (use $\alpha = 0.05$) that the mean heights in the two areas are equal. Also, compute the 95% confidence interval for the difference in population means. [5]

(c) A researcher wishes to test the claim that the average cost of tuition and fees of a two-year college is greater than \$5500. She selects a random sample of 36 two-year college and finds the mean to be \$5800. The population S.D. is \$600. Is there evidence to support the claim at $\alpha = 0.05$? Using the $p-$value method. [5]

8. (a) Distinguish between [6]
(i) Type I error and Type II error
(ii) Null hypothesis and alternative hypothesis
(iii) Rejection region and acceptance region

(b) A nutritionist claims that at most 65% of the preschool children in a certain country have a protein-deficient diet. A sample survey reveals that it is true for 244 children out of of a sample of 300 children. Is the nutritionist justified in his claim? Use a significant level of 1%. [4]

(c) A random sample of 27 pairs of observations from a normal population gave a correlation coefficient of 0.6. Is this significant of correlation in the population? [4]

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2^{nd} Year 1^{st} Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2109 Course Title: Introduction to Financial Mathematics

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Suppose you borrow 1,200 BDT at 4 percent compounded semiannually. The debt is amortized by making equal payments 200 BDT at the end of each six months. [Assume the interest rate provided here is the semiannual interest rate]. [10]
   - (i) After first payment, how much does it reduce the balance owed?
   - (ii) How much amount you have to pay at your last payment?
   - (iii) How much interest (total) you have to pay to amortized the debt?
   - (iv) How many years you need to amortized the debt?

   (b) A man wishes to have \$2,500 available in a bank account when his daughter's first-year college expenses begin. How much must he deposit now at 3.5 percent, compounded annually, if the girl is to start in college six years from now? [4]

2. (a) Write the formal definition of the following terms: [3]
   Exchange-Trade Markets, Bonds, Arbitrage Opportunity

   (b) Write down the difference between forward and future contract. [3]

   (c) Explain carefully the difference between hedging, speculation, and arbitrage. [3]

   (d) An investor buys a European put on a share for 3 BDT. The stock price is 42 BDT and the strike price is 40 BDT. Under what circumstances does the investor make a profit? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option. [5]

3. (a) A trader sells a put option with a strike price of \$35 for \$5. What is the trader's maximum gain or loss? How does your answer change if it is a call option. [3]

   (b) An investor sells a European call option with strike price of K and maturity T and buys a put with the same strike price and maturity. Describe the investor's position. [5]

   (c) A trader buys a call option with a strike price of \$45 and a put option with a strike price of \$40. Both options have the same maturity. The call costs \$3 and the put costs \$4. Draw a diagram showing the variation of the trader's profit with the asset price. [6]

4. (a) Define Brownian motion. Write down its probability density function mentioning its mean, variance and distribution. [3]

   (b) Suppose $B(t),\ t \geq 0$ be a standard Brownian motion. For any $0 < s,\ t < \infty$ evaluate the following: [6]
   - (i) Show that $E(B(t)B(s)) = \min\{s, t\}$.

(ii) Find $E\left(sB\left(\frac{1}{s}\right)tB(\left(\frac{1}{t}\right)\right)$.

(iii) Find $\frac{1}{c^2}E\left(B(c^2t)B(c^2s)\right)$; where c is a positive real constant.

(c) Show that Brownian motion path is nowhere differentiable. [5]

5. (a) Let $X_1,\ X_2, ..., X_n, ...$ be a sequence of independent identically distributed random variables with mean zero. If $S_n = X_1 + X_2 + ... + X_n$, then show that the process S is a discrete martingale. [5]

(b) Verify whether $B(t)^2$ is a martingale, where $B(t)$ represents a Brownian motion. [4]

(c) Show that the random process $\exp[B(t) - \frac{1}{2}t]$ is a martingale. [5]

6. (a) List six factors that affect stock option prices. [2]

(b) A 1-month European put option on a non-dividend-paying stock is currently selling for \$2.50. The stock price is \$47, the strike price is \$50, and the risk-free interest rate is 6% per annum. What opportunities are there for an arbitrageur? [5]

(c) Let P and C be a European Put and Call, with identical strike K and maturity T, written on a non-dividend paying asset S. Then show that [7]

$$C_t - P_t = S_t - Ke^{-r(T-t)}.$$

7. (a) Explain the no-arbitrage argument used to value an option when there is a one-step binomial tree. [3]

(b) A stock price is currently 20 BDT. It is known that at the end of 3 months it will be either 22 or 18 BDT. The risk-free rate of interest with continuous compounding is 12% per annum. Calculate the value of a 3-month European call option on the stock with an exercise price of 21 BDT. [5]

(c) Consider a European call option on a non-dividend-paying stock where the stock price is 40 BDT, the strike price is 40 BDT, the risk-free rate is 4% per annum, the volatility is 30% per annum, and the time to maturity is 6 months. (a) Calculate u, d, and p for a two-step tree. (b) Value the option using a two-step tree. [6]

8. (a) Considering $dS(t) = \mu S(t)dt + \sigma S(t)dB(t)$ derive the celebrated Black and Scholes partial differential equation. [4]

(b) For a portfolio with α shares of initial price $S(0) = S$; and a loan amount of β describe the martingale method of European call option pricing in one period binomial framework. [5]

(c) For a standard European call on an underlying with initial price $S(0)$, with maturity T and strike K, derive the famous Black-Scholes pricing expression $c(0) = S(0)N(d_1) - \exp(-rT)KN(d_2)$ for appropriate characterization of d_1 and d_2. [5]

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2^{nd} Year 1^{st} Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2111 Course Title: FORTRAN Programming

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Explain computer and computer generation. Sketch a block diagram of a typical computer. [4]

   (b) Write short note about machine language and high-level language. [4]

   (c) Define bits and bytes. Convert $(101.325)_{10}$ to its binary equivalent and $(1010101.00110)_2$ to its decimal equivalent. [6]

2. (a) Discuss the importance of Flowchart and Algorithm in FORTRAN program. [4]

   (b) Construct a flowchart and algorithm that will read two positive number m, n and return the value of $\beta(m, n)$. [5]

   (c) Write a FORTRAN program that will read n, print the first n; Fibonacci numbers and calculate average of them. [5]

3. (a) Explain logical operators. If $a = 4, b = 2, c = 6, d = 5$, then find the value of Y of the following expressions: [4]

$$Y = .\text{ NOT. } (a < b.\text{ AND. } c > d).EQV.(d > c.OR.b < c)$$

   (b) Write short note about Implicit none statement and character statement. [4]

   (c) Write FORTRAN expression for the following Mathematical statements: [6]

   i) $\sin^{-1}\left\{\log_{10}\left(|\sqrt{a} - b^2c|\right)^{\frac{1}{2}}\right\}$

   ii) $\left|\sqrt{x - y^3 - \dfrac{z^3}{\cos(a+b)}} + e^{-x^2}\right|$

   iii) $\dfrac{\sqrt[3]{a^2} + \sqrt{a^3}}{\sqrt{a\sqrt{b} + a}}$

4. (a) Write a flowchart and a FORTRAN program to solve the quadratic equation $Ax^2 + Bx + C = 0$. [7]

   (b) Write an IF-THEN-ELSE-END IF block that reads a temperature in degrees C and print out an appropriate message using the following criteria: [7]

Temperature $\leq 0°$C	Print "It's below freezing"
$0°$C $<$ Temperature $\leq 22°$C	Print "It's cold out"
$22°$C $<$ Temperature $\leq 28°$C	Prim "It's normal"
$28°$C $<$ Temperature $\leq 40°$C	Print "It's warm"
Temperature $> 40°$C	Print "It's hot!"

5. (a) Write a short note on SUBROUTINE and Function subprogram. Distinguish between SUBROUTINE and Function subprogram clearly. [7]

(b) Using SUBROUTINE write a FORTRAN program to sort $n = 20$ numbers in descending order. [7]

6. (a) Describe I, F, A, and X descriptors. [4]

(b) What will be printed out by each of the following FORTRAN statements? [4]

i REAL :: a = 1.602E − 19, b = 57.2957795, c = −1.
WRITE (*, '(ES14.7, 2(1X, E13.7))') a, b, c

ii REAL :: $a = -0.0001, b = 6.02 \times 10^{23}, c = 3.141593$
WRITE $(*, 20) a, b, c$
20 FORMAT (F10.3, 2X, E10.3, 2X, F10.5)

(c) Write a FORTRAN program to generate a table of square roots, squares, and cubes of first 20 natural numbers. [6]

7. (a) Describe array, rank of an array, and size of an array. [4]

(b) Write a FORTRAN program that will store the divisors of a number in an array. [4]

(c) Write a FORTRAN program that will take a 3×3 matrix B as input from the keyboard, then calculate and print the value of the transpose matrix of B. [6]

8. (a) Write a FORTRAN program to calculate $\beta(5,7)$ using function subprograrn that provides of $\Gamma(n)$ whenever it is called in the main program with a positive integer n. [7]

(b) For the following FORTRAN program (i) Draw the flowchart (ii) Write down the output [7]

```
INTEGER :: I
REAL :: A, B
B = 0
DO I = -5, 2
A = 3 * I + 5
IF (A .LE. 0) B = B + 1
END DO
PRINT*, B
END
```