Kishoreganj University

Department of Mathematics

2^{nd} Year 2^{nd} Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2201 Course Title: Linear Algebra II

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define Hermitian matrix.Prove that in a complex field every square matrix can uniquely be expressed as the sum of a Hermitian and a skew Hermitian matrix. [5]

   (b) Define Jordan block. Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$, calculate C such that $C^-1AC = J$. Hence calculate J. [6]

   (c) Verify that the matrix $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal. [3]

2. (a) Let W be a subset of a vector space V, then explain the symbol W^0. If $T : V \to U$ be linear and T^t its transpose, then write set $(\operatorname{Im} T)^0$ and $(\ker T)^0$. Prove that $\operatorname{Im} T^t = (\ker T)^0$. [5]

   (b) If W is a subspace of $\mathbb{R}^4$ generated by $(1, 1, 0, -1), (1, 2, 3, 0)$ and $(2, 3, 3, -1)$. Find a basis of W^0. [5]

   (c) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined by $f(x, y, z) = 3x - 2y$. Find $(T^t(f))(x, y, z)$, if $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x, y, z) = (x + y, y, y + z)$. [4]

3. (a) Define similar matrices and similarity invariants. Let T be the linear operator on $\mathbb{R}^2$ defined by [8]
$$T(x_1, x_2) = (4x_1 - x_2, 2x_1 + x_2)$$
   Consider the bases $E = \{(1, 0), (0, 1)\}$ and $S = \{(1, 3), (2, 5)\}$ for $\mathbb{R}^2$. Prove that the matrix representations $A = [T]_E$ and $B = [T]_S$ are similar. Hence verify that (i) $\det(A) = \det(B)$, (ii) $\operatorname{trace}(A) = \operatorname{trace}(B)$, and (iii) A and B have same eigenvalues.

   (b) What do you mean by the dual space of a vector space? Answer precisely. Let $V = \{a + bt \mid a, b \in \mathbb{R}\}$. Let $\varphi_1 : V \to \mathbb{R}$ and $\varphi_2 : V \to \mathbb{R}$ be defined by $\varphi_1(p(t)) = \int_0^1 p(t)dt$ and $\varphi_2(p(t)) = \int_0^2 p(t)dt$ respectively. Find a basis $\{v_1, \nu_2\}$ for V which is dual to $\{\varphi_1, \varphi_2\}$. [6]

4. (a) Define Inner product space and norm of vectors. Suppose that u, v, and w are vectors in an inner product space such that $< u, v >= 2, < v, w >= -6, < u, w >= -3, ||u|| = 1, ||v|| = 2$, and $||w|| = 7$. Evaluate the expression $< 2v - w, 3u + 2w >$. [5]

   (b) If $S = \{u_1, u_2, ..., u_n\}$ is an orthogonal set of nonzero vectors in an inner product space, then show that S is linearly independent and for any $v \in V$ the vector [5]
   $w = v- < v, u_1 > u_1- < v, u_2 > u_2 - ....- < v, u_n > u_n$
   is orthogonal to each of the u_i.

   (c) Suppose $p = x$ and $q = x^2$ are orthogonal with respect to the inner product $< p, q >= \int_{-1}^1 p(x)q(x)dx$. Show that $||p + q||^2 = ||p||^2 + ||q||^2$. [4]

5. (a) Explain the Gram-Schmidt process. Assume that the vector space $\mathbb{R}^3$ has Euclidean inner product, then transform the basis vector $\{u_1 = (1,1,1), u_2 = (0,1,1), u_3 = (0,0,1)\}$ into an orthogonal basis $\{v_1, v_2, v_3\}$ by using this Process. [5]

   (b) Define adjoint operator. Let S and T be the linear operator on V and let $k \in K$, then show that [5]
   $(i)(S+T)^* = S^* + T^*$
   $(ii)(ST)^* = T^*S^*$.

   (c) Find the least square straight line fit to the four points $(0,1), (1,3), (2,4)$, and $(3,4)$. [4]

6. (a) Define Bilinear form. Let F be the bilinear form on $\mathbb{R}^2$ defined by [7]
   $f((x_1,x_2),(y_1,y_2)) = 2x_1y_1 - 3x_1y_2 + x_2y_2$,
   Find the matrix A of f in the basis $\{u_1 = (1,0), u_2 = (1,1)\}$.

   (b) What is the congruent matrix? For a real symmetric matrix A, find a non-singular matrix P such that P^TAP is diagonal. Where [7]

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -1 \end{bmatrix}.$$

7. (a) What is called by singular value decomposition (SVD) of a $m \times n$ matrix? Construct a singular value decomposition of the matrix $A = \begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$. [7]

   (b) How do we define definite and semi- definite quadratic forms? Determine whether the quadratic form $2x_1^2 + 2x_2^2 + 3x_3^2 - 4x_2x_3 - 4x_3x_1 + 2x_1x_2$ is definite, semi-definite or indefinite. [7]

8. (a) Define saddle point. If $F(x,y) = 7 + 2(x+y)^2 - ysiny - x^3$ and $f(x,y) = 2x^2 + 4xy + y^2$, Does either $F(x,y)$ or $f(x,y)$ have a minimum value at $x = y = 0$. [5]

   (b) Write down the necessary and sufficient condition for a symmetric matrix A to be positive semidefinite and also show that [7]
   $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ is positive semiefinite.

   (c) If $f(x,y) = x^2 - 10xy + y^2$, show that f is not positive definite. [2]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

Department of Mathematics

2nd Year 2nd Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2202 Course Title: FORTRAN Programming Lab

Total Marks: 60 Time: 03.00 Hours Credits: 3

Answer all of the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Marks obtained in first-year final examination in different courses of a student are given in the following table [10]

Courses	MTH101	MTH102	MTH103	MTH104	MTH105	MTH106
Marks Obtained	81	65	73	77	57	39

Write a FORTRAN program to calculate the GPA for each course and then calculate the CGPA using the following grading system

Marks Obtained	80 -100	75 -79	70 -74	65 -69	60 -64	55 -59	50 -54	45 -49	40 -44	< 40
Grade Point	4.00	3.75	3.50	3.25	3.00	2.75	2.50	2.25	2.00	0.00

Input the obtained marks from the keyboard.

2. (a) Read a positive integer $N \geq 5$. Calculates to five decimal places of the sum and product: [5]

$$1, 1+\frac{1}{2}, 1+\frac{1}{2}+\frac{1}{3}, \cdots\cdots, 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{N}$$

(b) A man borrows Tk. 3000 from a bank at an interest rate of 7.5%. He pays 250 taka at the end of each month. Prints the amount he owes each month. Also, find the number of months/year that he must take to return the loan and the amount of his last installment. [5]

3. (a) The commissions of a salesman are as follows: [5]
i) If $0 < sales < $99, then the commission is 2% of the sales.
ii) If $100 ≤ sales < $299, then the commission is 5% of the sales.
iii) If $300 ≤ sales < $499, then the commission is 7.5% of the sales
iv) If sales ≥ $500, then the commission is 10% for the first $500 and 12.5% for the rest. Complete the following table by calculating the commissions of the salesman for the given sales.

Day	Sales	Commissions
Sunday	90	?
Monday	375	?
Tuesday	987	?
Wednesday	235	?
Thursday	500	?
Total Commissions		?

(b) A student takes N test on which scores range from 0 to 100 . Finds (i) the average score which is the average of the (N − 1) highest test scores, (ii) the number of rest marks which is less than 60 (iii) the number of test marks which is equal to 100. [5]

4. (a) Input an $N \times N$ matrix A into an input file. Test whether or not the given matrix A is a Nilpotent matrix. [5]

   (b) The password of your email address is az3dk1. Write a program to check whether your entered password to logon is valid or wrong. [5]

5. (a) From the following table of yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46,56 and 69 by using Newton's interpolation formula [10]

Age X	45	50	55	60	65	70
Premium Y	114.84	96.16	83.32	74.48	68.48	60.33

6. (a) Evaluate the definite integral $\int_0^1 \frac{1}{1+x^2} dx$ by using [10]
   (i) Simpson's 3/8 rule
   (ii) Romberg integration
   correct up to 5 decimal places. Compare your results with the exact value to identify the best rule of numerical integration.

Kishoreganj University

Department of Mathematics

2^{nd} Year 2^{nd} Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2203 Course Title: Integral and Vector Calculus

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define Iterated Integrals. Sketch the region of integration for the integral [5]

$$\int_0^2 \int_{x^2}^{2x} (4x+2)\,dy\,dx$$

and write an equivalent integral with the order of integration reversed and hence evaluate it.

(b) State Fubini's Theorem (Stronger form) for double integrals and hence evaluate [5]

$$\iint_D (x+2y)\,dA,$$

where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.

(c) Find the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis, and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x, y) = 3 - x - y$. [4]

2. (a) Define simple polar region with an example. How to find the limits of integration of $f(r, \theta)$ over the region R that inside the cardioid $\mathrm{r} = 1 + \cos\theta$ and outside the circle $\mathrm{r} = 1$. [4]

(b) Estimate $\iint\limits_R \sin\theta\,dA$ where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1 + \cos\theta)$. [4]

(c) A thin plate covers the triangular region bounded by the x-axis and the lines $x = 1$ and $y = 2x$ in the first quadrant. If the plate's density is $\delta(x, y) = (6x + 6y + 6)$ then find the center of mass and moments of inertia about the coordinate axes. [6]

3. (a) Define centroid of an object. Find the centroid of the solid enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the paraboloid $z = x^2 + y^2$ and bounded below by the xy-plane. [7]

(b) Find the volume of the solid enclosed between the paraboloids [7]

$$z = 5x^2 + 5y^2 \text{ and } z = 6 - 7x^2 - y^2.$$

4. (a) Utilize cylindrical coordinates to evaluate the integral [8]

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} x^2\,dv.$$

(b) Use spherical coordinates to find the volume of the solid G bounded above by the sphere $x^2 + y^2 + z^2 = 16$ and below by the cone $z = \sqrt{x^2 + y^2}$. [6]

5. (a) Evaluate $\iiint\limits_G xyz dv$, where G is the solid in the first octant is bounded by the parabolic cylinder $z = 2 - x^2$ and the planes $z = 0,\ y = x,\ y = 0$. [7]

   (b) Define conservative field. Show that, $\vec{F}$ is a conservative field if and only if [7]

$$\vec{\nabla} \times \vec{F} = \mathbf{0}.$$

6. (a) If $\mathbf{A} = (2y + 3)\mathbf{i} + xz\mathbf{j} + (yz - x)\mathbf{k}$, then show that $\int_c \mathbf{A}\cdot d\mathbf{r} = 8$ along the path c, the straight line joining $(0, 0, 0)$ and $(2, 1, 1)$. [7]

   (b) Show that $\iint\limits_S \phi \mathbf{n} ds = 100(\mathbf{i} + \mathbf{j})$, where $\phi = \frac{3}{8}xyz$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. [7]

7. (a) State and prove Green's Theorem in a plane. [7]

   (b) Verify Green's theorem in the plane for $\oint_C (xy + y^2)\, dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. [7]

8. (a) Prove [6]

$$\iiint \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) dV = \iint (\phi\nabla\psi - \psi\nabla\phi) \cdot d\mathbf{S}$$

   (b) State Stoke's theorem. Verify the Stokes's theorem for $\mathbf{A} = x^2\mathbf{i} + xy\mathbf{j}$ taken over the region in xy plane bounded by the lines $x = 0,\ y = 0,\ x = a,\ y = a$. [8]

Kishoreganj University

Department of Mathematics

2^{nd} Year 2^{nd} Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2105 Course Title: Numerical Analysis-I

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) State Intermediate Value Theorem. Describe the Bisection method for finding a root of the function $y = f(x)$ in the interval $[a, b]$. [4]

   (b) Write the algorithm for the Newton-Raphson method for finding a root of the function $y = f(x)$ in the interval $[a, b]$. [4]

   (c) Approximate the root of $f(x) = \cos(x) + 2\sin(x) + x^2$ with the Newton's method starting with $x_0 = 0.01$ and use tolerances' $\varepsilon_f = 0.002, \varepsilon_{ab} = 0.001$. Consider four decimal digits arithmetic to find the solution. [6]

2. (a) Discuss the fixed-point iteration method for finding a real root of the equation $f(x) = 0$ in $[a, b]$. What is the condition of convergence of the method ? [6]

   (b) Find the root of the equation $x^2 - \ln x - 2 = 0$ on [1,2] by Fixed-point method correct up to three decimal places, take $x_0 = 1.0$. [5]

   (c) Write three situations where Newton's method may not converge quickly. [3]

3. (a) Define interpolation and extrapolation. Derive Lagrange's interpolating polynomial formula for unequally spaced x-values. [6]

   (b) Given points $(x, f(x))$ as $(1, 1), (2, 8), (3, 27), (4, 64), (5, 125), (6, 216), (7, 343)$ and $(8, 512)$. [8]
   i) Use Newton's Forward difference formula to find $f(2.5)$.
   ii) Use Newton's Backward difference formula to find $f(7.5)$.

4. (a) Derive Newton's divided difference interpolation formula for polynomial interpolation with unequal intervals. [7]

   (b) Given [7]

x	0	1	3	4	6
$f(x)$	-12	0	12	24	90

   Use the above formula in 4(a) to evaluate the interpolating polynomial in standard form.

5. (a) Given a set of $(n+1)$ points $\{(x_0, y_0), (x_1, y_1), (x_2, y_2), \cdots\cdots, (x_n, y_n)\}$, derive a $3-$ point formula to compute $f'(x)$ for any value of x in $[x_0, x_n]$. [7]

   (b) Form a data table for the function $f(x) = xe^x$ with the three x-values $2.0, 2.1$ and 2.2 and then compute $f'(2.1)$ and $f''(2.1)$ by using the 3-point formula. Then compare the numerical results with the exact values. [7]

6. (a) State Gaussian quadrature theorem. In Gaussian quadrature, how can we change the arbitrary interval $[a, b]$ to $[-1, 1]$, also show graphically. [4]

   (b) Use the Composite Trapezoidal rule to find approximations to $\int_0^{\pi} \sin x dx$ with $n = 1, 2, 4$. Then perform Romberg integration on the results. [5]

   (c) Given, $\int_0^{\pi/2} \sin x dx = 1$. Using adaptive quadrature formula, compute $S\left(0, \frac{\pi}{2}\right), S\left(0, \frac{\pi}{4}\right) \& S\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Also verify the error estimate $\frac{1}{15}\left|S(a, b) - S\left(a, \frac{a+b}{2}\right) - S\left(\frac{a+b}{2}, b\right)\right| < \varepsilon$ for this problem. [5]

7. (a) Solve the following system of linear equations by using Gaussian elimination method (partial pivoting) with 3 -digit rounding arithmetic [4]

$$\begin{aligned} x + y + z &= 1 \\ 3x + y - 3z &= 5 \\ x - 2y - 5z &= 10 \end{aligned}$$

   (b) Consider the linear system [5]

$$\begin{aligned} 2x + 3y + z &= 9 \\ x + 2y + 3z &= 6 \\ 3x + y + 2z &= 8 \end{aligned}$$

   Use LU factorization scheme to solve the system.

   (c) Solve the linear system [5]

$$\begin{aligned} 6x + y + z &= 20 \\ x + 4y - z &= 6 \\ x - y + 5z &= 7 \end{aligned}$$

   using Jacobi method to find the first four iterative values with $\underline{x}^{(0)} = \underline{0}$.

8. (a) What is Iterative method? Describe the SOR iterative technique for solving the system of linear equations $A\underline{x} = \underline{b}$. [7]

   (b) Solve the following linear system by SOR method to find the first two iterative values with $\underline{x}^{(0)} = \underline{0}$ and $\omega = 1.1$, where [7]

$$\begin{aligned} 10x - y &= 9 \\ -x + 10y - 2z &= 7 \\ -2y + 10z &= 8 \end{aligned}$$

Kishoreganj University

Department of Mathematics

2^{nd} Year 2^{nd} Semester B.Sc. (Honours) Final Examination-2023

Course Code: MAT 2207 Course Title: Discrete Mathematics

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) What is Tautology and, Logically equivalent? Prove that the converse and the inverse of a conditional statement are equivalent. [5]

   (b) What is universal and existential quantification? What is the truth value of $\forall x(x^2 \geq x)$ if the domain consists of all real numbers? What is the truth value of this statement if the domain consists of all integers? [5]

   (c) What is Universal instantiation? Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job." [4]

2. (a) What is proof by contraposition? Prove that if n = ab, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$. [5]

   (b) Translate the statement "The sum of the two positive integers is always positive" into a logical expression. [3]

   (c) Prove that "If n is an integer and $3n + 2$ is odd, then n is odd", using both the contraposition and contradiction theorem. [6]

3. (a) What is the Boolean function? Find the values of the Boolean function represented by $F(x, y, z) = xy + \overline{z}$. [4]

   (b) Contract the circuits that produce the following outputs: [6]
   $(i)(x + y)\overline{x}$,
   $(ii)\overline{x}(\overline{y + \overline{z}})$ and
   $(iii)(x + y + z)(\overline{x}\ \overline{y}\ \overline{z})$.

   (c) By using Karnaugh maps, minimize these sum of products expansion [4]
   $(i)x\overline{y}z + x\overline{y}\ \overline{z} + \overline{x}yz + \overline{x}\ \overline{y}z + \overline{x}\ \overline{y}\ \overline{z}$
   $(ii)xyz + xy\overline{z} + x\overline{y}z + x\overline{y}\ \overline{z} + \overline{x}yz + \overline{x}\ \overline{y}z + \overline{x}\ \overline{y}\ \overline{z}$.

4. (a) Prove the following theorem. [8]
   Let c_1 and c_2 be real numbers. Suppose that r^2 - c_1r - c_2 = 0 has two distinct roots r_1 and r_2. Then the sequence a_n is a solution of the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ if and only if $a_n = \alpha_1 {r_1}^n + \alpha_2 {r_2}^n$ for n = 0,1,2,..., where α_1 and α_2 are constants.

   (b) What is the solution of the recurrence relations [6]

      i. $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$.

      ii. $a_n = 2a_{n-1}$ with $a_0 = 3$.

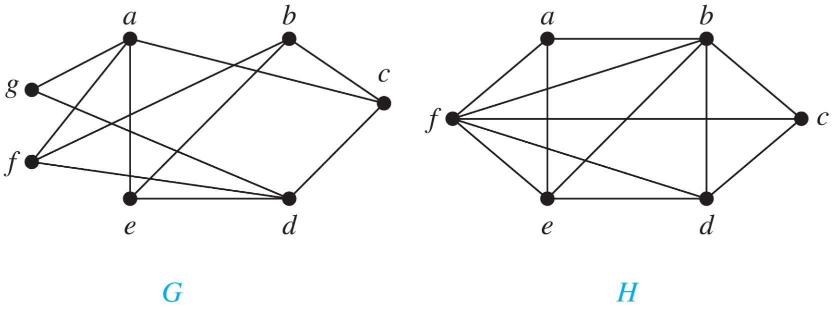
5. (a) Find all solutions of the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$. [4]

(b) What is generating function? Use generating functions to find an explicit formula for a_n, where $a_n = 8a_{n-1} + 10^{n-1}$, $a_1 = 9$. [6]

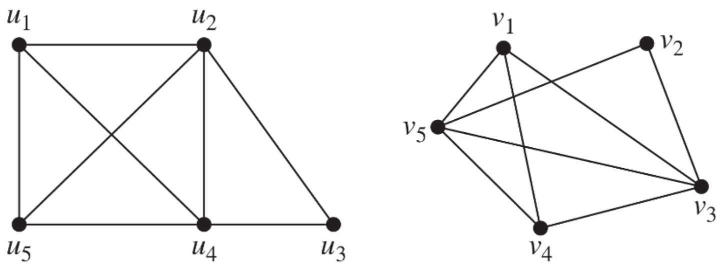
(c) During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. [4]

6. (a) What is complete graphs, cycles and wheels, explain with example. [3]

(b) What is bipartite graphs? Are the graphs G and H bipartite? [5]



(c) What is graph isomorphism? Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists. [6]



7. (a) Define connected and disconnected graph. Show that there is a small path between every pair of distinct vertices of of a connected undirected graph. [6]

(b) What are the degrees and the neighborhoods of the vertices in the graphs G and H displayed in the following figure? [4]

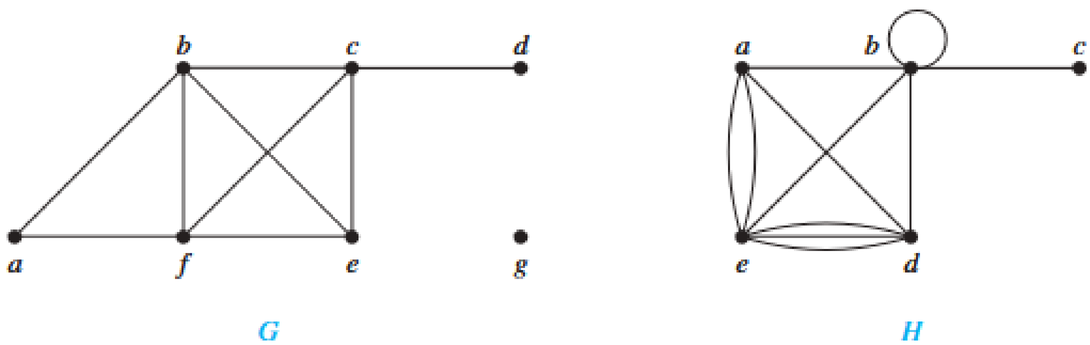


Figure 1: The undirected graphs G and H.

(c) Find the length of the shortest path between a and z in the weighted graph shown in the following figure. [4]

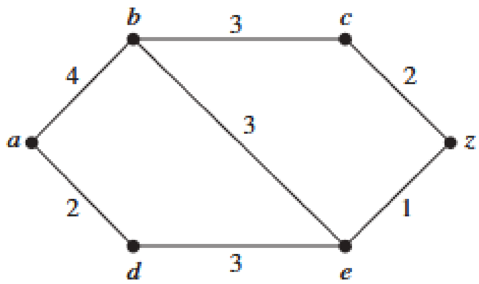


Figure 2: A weighted simple graph.

8. (a) Prove that, a tree with n vertices has n-1 edges. [4]

   (b) Form a binary search tree for the words mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry (using alphabetical order). [6]

   (c) Use Prim's algorithm to find a minimum spanning tree in the following graph. [4]