Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 1st Semester B.Sc (Hons.) Final Examination-2023
Department of Mathematics
MAT 1101: Fundamentals of Mathematics (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define injective, surjective, and inverse functions. If $f : A \to B$ be a function which is defined by $f(x) = \frac{x-3}{2x+1}$, where $A = \mathbb{R} - \left\{-\frac{1}{2}\right\}$ and $B = \mathbb{R} - \left\{\frac{1}{2}\right\}$. Prove that f is injective and surjective functions. Also find f^{-1}. [6]

   (b) Explain equivalence relation and partial order relation. Show that the relation 'congruence modulo **m**' is an equivalence relation in the set of integers. [4]

   (c) By using Venn Diagrams prove that, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for all sets A, B, and C. [4]

2. (a) Define tautology and fallacy of a statement. Show that [6]
   (i) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ is a tautology and
   (ii) $(p \wedge q) \wedge \sim (p \vee q)$ is a fallacy.

   (b) Define symbolic argument. Write down some standard forms of valid and invalid arguments, and determine the validity of the following argument: [4]
   The grass is green or the grass is full of weeds.
   The grass is not green.
   ___
   ∴ The grass is full of weeds.

   (c) If p, q and r are statement such that $p \Rightarrow q$ and $q \Rightarrow r$ then prove that $p \Rightarrow r.$ [4]

3. (a) State De Moivre's theorem. Using this theorem, solve the following equation [6]

$$x^7 + x^4 + x^3 + 1 = 0.$$

   (b) Represent graphically the set of values of the complex number z for which $\left|\frac{z-3}{z+3}\right| > 2$. [4]

   (c) If $x = \cos\theta + i\sin\theta$ and $1 + \sqrt{1 - a^2} = na$, then prove that [4]

$$1 + a\cos\theta = \frac{a}{2n}(1 + nx)\left(1 + \frac{n}{x}\right)$$

.

4. (a) Determine the general term and the sum to n terms of the series [6]

$$2 + 12 + 36 + 80 + 150 + 252 + \cdots\cdots$$

   (b) Find the sum to n terms of the series [4]

$$\tan^{-1}\alpha + \tan^{-1}\frac{1}{1 + 1.2\alpha^2} + \tan^{-1}\frac{1}{1 + 2.3\alpha^2} + \cdots\cdots$$

   (c) Evaluate the sum to infinity of the series [4]

$$\frac{3}{1.2.4} + \frac{4}{2.3.5} + \frac{5}{3.4.6} + \cdots\cdots$$

5. (a) For any number of unequal positive quantities prove that $\boldsymbol{A.M > G.M > H.M}$. [6]

   (b) Prove that $\boldsymbol{abcd > 81(s-a)(s-b)(s-c)(s-d)}$ if $\boldsymbol{a+b+c+d = 3s}$ and all the factors are positive. [4]

   (c) If $\boldsymbol{s = a_1 + a_2 + \cdots + a_n}$ and $\boldsymbol{a_1, a_2, \cdots, a_n}$ are not equal, show that [4]

$$\frac{s}{s-a_1} + \frac{s}{s-a_2} + \cdots + \frac{s}{s-a_n} > \frac{n^2}{n-1}$$

.

6. (a) State and prove Cauchy's inequality. [6]

   (b) If $\boldsymbol{x, y, z}$ are positive real numbers such that $\boldsymbol{x^2 + y^2 + z^2 = 27}$, then by applying the above theorem show that $\boldsymbol{x^3 + y^3 + z^3 \geq 81}$. [4]

   (c) If $\boldsymbol{n}$ is a positive integer and $\boldsymbol{0 < a < 1}$, prove that (i) $\boldsymbol{1 + na < (1+a)^n < \frac{1}{1-na}}$, and (ii) $\boldsymbol{1 - na < (1-a)^n < \frac{1}{1+na}}$. [4]

7. (a) State Descartes's rule of sign. Find the nature of the roots of the equation [6]

$$6x^4 - 25x^3 + 81x^2 - 9x - 13 = 0.$$

   (b) Let $\boldsymbol{\alpha, \beta,}$ and $\boldsymbol{\gamma}$ are the roots of a cubic equation with principal coefficient **1** and $\boldsymbol{\sum \alpha = 1, \sum \alpha^2 = 3}$ and $\boldsymbol{\sum \alpha^3 = 7.}$ Find the value of $\boldsymbol{\sum \alpha^7.}$ [4]

   (c) Find the equation whose roots are the roots of the equation $\boldsymbol{4x^5 - 2x^3 + 7x - 3 = 0}$, each increased by 2 . [4]

8. (a) Four students are running for president of the Honor Society: Antoine (A), Betty (B), Camille (C), and Don (D). The club members were asked to rank all candidates. The resulting preference table for this election is given in the following Table. [7]

Honor Society Preference Table					
Numbers of Votes	19	15	11	7	2
First	B	C	D	A	C
Second	A	A	C	D	D
Third	C	D	A	C	A
Fourth	D	B	B	B	B

   Use the Borda count method and the pairwise comparison method to determine the winner of the election for president of the Honor Society.

   (b) The Republic of Geranium needs to apportion **250** seats in the legislature. Suppose the population is **8, 800, 000** and there are five states, $\boldsymbol{A, B, C, D}$, and $\boldsymbol{E}$. The **250** seats are to be divided among the five states according to their respective populations, given in the following table. [7]

Republic of Geranium Population					
State	A	B	C	D	E
Population	1003200	1228600	4990700	813000	764500

   Use Hamilton's method and Webster's method to apportion the seats.

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1^{st} Year 1^{st} Semester B.Sc (Hons.) Final Examination-2023
Department of Mathematics
MAT 1103: Differential Calculus (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define domain and range. Find the domain and range of the following functions (any three): (i) $f(x) = \dfrac{1}{(x-1)(x-4)}$, (ii) $g(x) = \sqrt{x^2 - 9}$, (iii) $h(x) = \operatorname{sech} x$, (iv) $p(x) = \dfrac{|x-3|}{x-3}$ [7]

   (b) If $f(x) = x^2 - 4x + 3$ then sketch $f(-x), |f(x)|, f(|x|)$, and $f(2x)$. [4]

   (c) Determine all horizontal and vertical asymptotes for $f(x) = \dfrac{1}{x^2 - 4}$. [3]

2. (a) Given that $7x \le f(x) \le 3x^2 + 2$ for all x, determine the value of $\lim\limits_{x \to 2} f(x)$. [4]

   (b) A function $f(x)$ is defined as follows: $f(x) = \begin{cases} e^{-|x|/2} & \text{when } -1 < x < 0 \\ x^2 & \text{when } 0 \le x < 2 \end{cases}$ [5]
   Does $\lim\limits_{x \to 0} f(x)$ exists?

   (c) By $(\delta - \epsilon)$ definition, show that $\lim\limits_{x \to 1} \dfrac{x-1}{x^2 + 2x - 3} = \dfrac{1}{4}$. [5]

3. (a) Define the continuity and differentiability of a function $f(x)$ at a point $x = a$. What do you know about different types of discontinuity? [4]

   (b) Discuss the differentiability and continuity of the function $f(x)$ at $x = \dfrac{\pi}{2}$, [5]

   $$\text{where} \quad f(x) = \begin{cases} 1, & \text{if } x < 0 \\ 1 + \sin x, & \text{if } 0 \le x < \dfrac{\pi}{2} \\ 2 + \left(x - \dfrac{\pi}{2}\right)^2, & \text{if } x \ge \dfrac{\pi}{2} \end{cases}$$

   (c) The function $f(x)$ is defined as follows. [5]

x	-3	-2	-1	0	1	2	3
$f(x)$	-26	-7	0	1	2	9	28

   Evaluate $y_1 = f(x) - 2$, $y_2 = -f(x)$, $y_3 = f(-x)$, $y_4 = f^{-1}(x)$, and $y_5 = f(|x|)$.

4. (a) State **Intermediate Value Theorem.** Use this theorem, show that the equation $x^3 + x^2 - 2x = 1$ has at least one root in the interval $[-1, 1]$. [4]

(b) Use L Hospital's rule to evaluate any two of the following: [6]

$$(i)\lim_{x\to 0}\frac{\tan x - x}{x^3}\quad (ii)\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\tan x - 1}\quad (iii)\lim_{x\to 1}\left(\frac{1}{\log x}-\frac{1}{x-1}\right)$$

(c) If $F(x) = f(x\,g(x))$ then find the value of $F'(1)$ where $f'(1) = 1, g(1) = 1, g'(1) = 1.$ [4]

5. (a) If $\ln y = \operatorname{arc}\tan x$, then show that [4]

$$(1+x^2)\,y_{n+2}+(2nx+2x-1)y_{n+1}+n(n+1)y_n=0.$$

(b) Take into account $y = e^{ax}\cos(bx+c)$, prove that [4]

$$y_n=(a^2+b^2)^{n/2}\,e^{ax}\cos\left(bx+c+n\operatorname{arc}\tan\frac{b}{a}\right).$$

(c) The volume of a spherical ballon is increasing at the rate of 12 cm^3/sec. Find the rate of change of its surface at the instant when its radius is 6 cm. [2]

(d) Differentiate $\tan^{-1}\dfrac{2x}{1-x^2}$ with respect to $\sin^{-1}\dfrac{2x}{1+x^2}$. [4]

6. (a) Justify Rolle's Theorem for the function $f(x) = x^{2/3}$, when $x \in (-1, 1)$. [4]

(b) Take into account a function [4]

$$f(x)=\begin{cases} x\sin\dfrac{1}{x} & \text{for}\quad -1<x<1 \ \text{ and } \ x\neq 0\\ 0 & \text{for}\quad x=0\end{cases}$$

Justify the given function for the Mean Value Theorem in the interval $(-1, 1)$.

(c) Determine the extremum of $f(x) = 5x^6 - 18x^5 + 15x^4 - 10$. [6]

7. (a) Find the interval where the function $f(x) = x^3 - 3x^2 + 2x - 4$ is concave down and concave up also find the inflection point if exists. [6]

(b) For a given curved surface of a right circular cone when the value is maximum, show that the semivertial angle is $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$. [8]

8. (a) Expand $e^{ax}\cos bx$ in finite Maclaurin's series with the remainder in Lagrange's form. [6]

(b) Use the root test to determine whether the following series converge or diverge [4]

(i) $\sum_{k=1}^{\infty}\left(\dfrac{3k+2}{2k-1}\right)^k$.

(ii) $\sum_{k=1}^{\infty}\dfrac{1}{\{\ln(k+1)\}^k}$.

(c) Use Taylor's theorem to expand $f(x) = \dfrac{1}{x+1}$ in power of $(x-3)$. [4]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1^{st} Year 1^{st} Semester B.Sc (Hons.) Final Examination-2023
Department of Mathematics
MAT 1105: Analytic Geometry (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Identify and graph the equations: [4]
   (i) $r = 3.$
   (ii) $\theta = \frac{\pi}{4}.$

   (b) In what ratio is the straight line joining the points $(3, 4)$ and $(8, 1)$ divided by the $x-$ axis? Find the abscissa of this point on the $x-$ axis. [2]

   (c) Transform the Polar equations into Cartesian equations: [4]
   (i) $x^2 + y^2 = e^2(p + x)^2$
   (ii) $r(\cos 3\theta + \sin 3\theta) = 5k \sin\theta \cos\theta.$

   (d) Transform the Cartesian equations into Polar equations: [4]
   (i) $\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{r^2}$
   (ii) $x^4 + x^2y^2 - (x + y)^2 = 0.$

2. (a) The direction of axes remaining the same, choose a new origin such that the new coordinates of the pair of points whose old coordinates are $(5, -13)$ and $(-3, 11)$ may be the form (h, k) and $(-h, -k)$. [4]

   (b) Transform the equation $17x^2 + 18xy - 7y^2 - 16x - 32y - 18 = 0$ to one in which there is no term involving x, y and xy both sets of axes being rectangular. [6]

   (c) By transforming to parallel axes through a properly chosen point (h, k), prove that the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$ can be reduced to one containing only the terms of the 2nd degree. [4]

3. (a) Find the condition that the general equation of the second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a pair of straight lines. [6]

   (b) Prove the equation $y^3 - x^3 + 3xy(y - x) = 0$, represents three straight lines equally inclined to one another. [6]

   (c) If the pair of straight lines $x^2 - 2axy - y^2 = 0$ and $x^2 - 2bxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, prove that $ab = -1.$ [2]

4. (a) Prove that the straight lines represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ will be equidistant from the origin, if $f^4 - g^4 = c(bf^2 - ag^2)$. [7]

   (b) If two straight lines represented by the equation $x^2(\tan^2\psi + \cos^2\psi) - 2xy\tan\psi + y^2\sin^2\psi = 0$ makes an angle α and β with $x-$ axis respectively, then show that $\tan\alpha - \tan\beta = 2.$ [7]

5. (a) Two Loran stations located a station $\mathbf{200}$ miles apart along a straight shore. If a ship record a time difference of $\mathbf{0.00043}$ second between the Loran signals and continue on the hyperbolic path corresponding to the difference. (The radio signals travel $\mathbf{1,86,000}$ miles per second.) Where does it reach shore? If the ship is $\mathbf{50}$ miles offshore, what is the position of the ship? [6]

   (b) For the equation $31x^2 + 10\sqrt{3}xy + 21y^2 - 32x + 32\sqrt{3}y - 80 = 0.$ [8]
   (i) Determine which conic the equation represents.
   (ii) Find the rotation angle required to eliminate the xy - term ordinate system.
   (iii) Transform the equation into its standard form.
   (iv) Draw the graph of the resulting conic.

6. (a) $A,\ B,\ C$ are three points on the axes $x,\ y$ and z respectively at distance $a,\ b,\ c$ from origin O, find the co-ordinates of the point which is equidistant from $A,\ B,\ C,$ and O. [4]

   (b) The direction cosines of a moving line in two adjacent positions are $l,\ m,\ n$ and $l+\delta,\ m+\delta,\ n+\delta.$ Show that the small angle $\delta\theta$ between the positions is given by $(\delta\theta)^2 = (\delta l)^2 + (\delta m)^2 + (\delta n)^2$. [4]

   (c) If $l_1,\ m_1,\ n_1$ and $l_2,\ m_2,\ n_2$ be the direction cosines of any two lines AB and CD and θ be the angle between them. Then prove that $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$. [6]

7. (a) Find the equation of the plane through the points $(2,2,1)$ and $(9,3,6)$ and perpendicular to the plane $2x + 6y + 6z = 9.$ [4]

   (b) Determine the equation of the plane passing through the line of intersection of the planes $2x - y = 0$ and $3z - y = 0$ and perpendicular to the plane $4x + 5y - 3z + 7 = 0.$ [4]

   (c) Two system of rectangular axes have the same origin. If a plane cuts the axes at $a,\ b,\ c$ and $a_1,\ b_1,\ c_1$ respectively from the origin. Prove that [6]

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a_1^2} + \frac{1}{b_1^2} + \frac{1}{c_1^2}.$$

8. (a) Find the length and equation of shortest distance line between the two lines [7]

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}.$$

   (b) Find the equation of a planes which bisect the planes $3x - 2y + 6z + 8 = 0$ and $2x - y + 2z + 3 = 0.$ [7]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 1st Semester B.Sc (Hons.) Final Examination-2023

Department of Mathematics

STA 1107: Basic Statistics (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define Statistics. How was it originated? Make a brief review of various definitions of statistics and in the light of these definitions; give your own definition. [4]

   (b) Discuss the reasons why the scope of statistics has grown so tremendously in recent years. [3]

   (c) The data below specify the longevity of some electric bulbs in months. [7]
   3.4, 3.1, 3.7, 4.4, 3.2, 3.2, 2.2, 4.1, 3.5, 4.5, 3.3, 3.2, 3.7, 3.0, 2.6, 3.0, 1.1, 5.1, 4.9, 3.4, 2.5, 1.5, 4.3, 3.1, 3.4, 3.6, 3.8, 3.1, 4.7, 3.7, 2.9, 3.3, 3.9, 3.1
   (i) Taking a suitable class interval, construct a frequency distribution of the data.
   (ii) Then draw the Ogive curve, and also indicate the median on the diagram.

2. (a) What are quartiles, deciles and percentiles? What role do they play in statistics as measures of central tendency? [4]

   (b) The variables $\boldsymbol{X}$ and $\boldsymbol{Y}$ are related by the equation $\boldsymbol{Y = 3X + 4}$. If the mean value of $\boldsymbol{X}$ is 19 , find the mean value of $\mathbf{Y}$. Also, determine the value of $\boldsymbol{k}$ for which $\boldsymbol{\sum_{i=1}^{n} (x_i - k) = 0}$ [4]

   (c) The following values are the total times in minutes that 16 battery packs operated before requiring recharge: [6]

814	793	775	746	758	729	749	760
736	804	764	756	778	728	745	780

   Determine the third quartile and seventh decile for these values. Obtain the percentile rank corresponding to the value 764 and interpret your result.

3. (a) What is Standard Deviation? State its uses. Examine the effect of change of origin and scale of measurement on Standard Deviation. [5]

   (b) Calculate the coefficient of variance for the following distribution. [5]

Class	4.5-12.5	12.5-20.5	20.5-28.5	28.5-36.5	36.5-44.5	44.5-52.5
Frequency	4	24	21	18	5	3

   (c) Discuss the importance of mean, median and mode in statistical research. Also, explain the advantages and disadvantages of one over the others. [4]

4. (a) What is kurtosis? What does it measure? How does the measure of kurtosis help in understanding a frequency distribution? [4]

(b) Show that $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$ are invariant to the chances in origin and scale of measurement. [4]

(c) The first four moments of a distribution about 4 were found to be $\mathbf{-1.5, 17,}$ $\mathbf{-30}$ and 108. Calculate the corresponding central moments and hence find $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$ and comment on the skewness and kurtosis of the distribution. [6]

5. (a) Derive the formula for Spearman's rank correlation coefficient. What are the main differences between Pearson's correlation coefficient and Spearman's rank correlation coefficient? [5]

(b) Explain the partitioning of the total variation in regression. Also, explain the goodness of fit in regression. [5]

(c) The correlation coefficient for the data $\boldsymbol{y_1, y_2, y_3}$ are $\boldsymbol{r_{13} = 0.567}$, $\boldsymbol{r_{23} = 0.895}$ and $\boldsymbol{r_{12} = 0.335}$. Compute the multiple correlation coefficient $\boldsymbol{R_{3.12}}$ and partial correlation coefficient $\boldsymbol{r_{23.1}}$ [4]

6. (a) What does a coefficient of correlation measure? [5]
Discuss the situation with sketch when $\boldsymbol{r = +1, r = -1, r = 0}$.

(b) Two variables x and y have the following Bi-variate distribution. [9]

	y values				
x values	7	10	13	16	19
1.5	3	2	3	7	2
3.5	3	2	3	3	1
5.5	-	-	1	-	-
7.5	1	3	1	-	1
9.5	-	-	-	1	-

Calculate the correlation coefficient of the Bi-variate data.

7. (a) Distinguish between (i) Linear and curvilinear regressions and (ii) Curvilinear and polynomial regressions. [4]

(b) Explain the properties of a simple regression line mathematically. [5]

(c) The following data (in hours) indicate the training time and the time to complete a project of the six different students: [5]

Training time	22	19	24	18	20	21
Project complete time	13	17	14	17	16	18

Estimate the Spearman's rank correlation coefficient.

8. The accompanying table shows the proportions of coal miners who exhibit symptoms of pneumoconiosis to their number of years of working in coal mines. [14]

Years	5	10	15	20	25	30	35	40	45	50
Proportions	0	.01	.02	.07	.15	.17	.18	.21	.35	.45

i) Calculate the regression line of proportion with pneumoconiosis (y) on working years $(\boldsymbol{x})$.
ii) Obtain the standard error of the estimate and hence find the coefficient of correlation between $\boldsymbol{x}$ and $\boldsymbol{y}$.
iii) Calculate the correlation coefficient directly from the formula and compare the same with the one obtained in (ii).
iv) Use your fitted regression line to estimate the proportion of coal miners developing pneumoconiosis who have worked for 42 years and 52 years.

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1st Year 1st Semester B. Sc. (Hons.) Final Examination-2023

Department of Mathematics

PHY 1109: Mechanics, Waves and Properties of Matters (3 Credits)

Time: 3 Hour **Full Marks: 70**

Answer any **FIVE (5)** questions of the following **EIGHT (8)** questions.

1. a) Consider two billiard balls *a* and *b* of identical masses *m* shown in Fig. 01. Ball *a* approaches the stationary ball *b* at a velocity along the positive x direction. After an elastic collision, the balls *a* and *b* move away at angles ξ and δ, respectively, with X-axis as shown in the figure. Determine the final speeds of the balls and determine the angle between the lines along which they escape the collision region. **7**

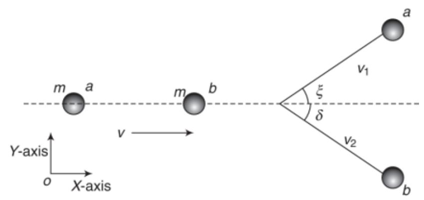

Fig. 01

1. b) Frogs can stretch their muscles and leap as much as over 10 meters, more than 20 times their length as shown in Fig. 02. A frog at point A is trying to jump from the point *A* to *B* as shown in the figure. The horizontal distance between the frog and the step is *L*, and the point *B* is at a height *h*. The angle that the line *AB* makes with the horizontal line is β. If the frog jumps out an angle α with the horizontal with an initial speed v_0, determine the angle α in terms of the angle β such that the frog expends minimum energy to reach point *B*. **7**

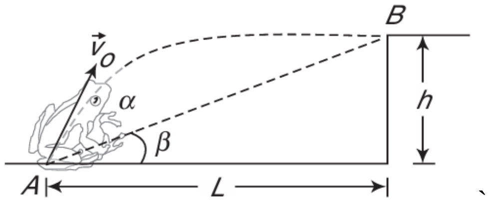

Fig. 02

2. a) A robotic lander with an earth weight of 3430 Newton is sent to Mars, which has radius $R_M = 3.39\ 10^6$ meters and mass m = 6.42×10^{23} kg. Find the weight F_g of the lander on the Martian surface and the acceleration there due to gravity, g. **4**

2. b) Derive the expression for gravitational potential energy using the gravitational force equation and the concept of work done against gravity. **4**

2. c) Describe Kepler's Laws of Planetary Motion and explain how they relate to the motion of planets within a solar system. Provide detailed explanations for each of Kepler's laws and discuss their significance in understanding the dynamics of celestial bodies. **6**

3. a) Explain the concept of energy in simple harmonic motion (SHM) and derive the equations for kinetic energy, potential energy, and total mechanical energy of an object undergoing SHM. Discuss how these energies change as the object oscillates back and forth around its equilibrium position. **4**

3. b) Describe the different types of mechanical waves and provide examples of each type. Explain the characteristics that distinguish longitudinal waves from transverse waves, including the direction of particle displacement and wave propagation. **4**

3. c) Cousin Throckmorton holds one end of the clothesline taut and wiggles it up and down sinusoidally with frequency 2.00 Hz and amplitude 0.075 m. The wave speed on the clothesline is $v = 12.0$ m/s. At $t = 0$ Throcky's end has maximum positive displacement and is instantaneously at rest. Assume that no wave bounces back from the far end. (i) Find the wave amplitude A, angular frequency v, period T, wavelength λ , and wave number k. (ii) Write a wave function describing the wave. (iii) Write equations for the displace-ment, as a function of time, of Throcky's end of the clothesline and of a point 3.00 m from that end. **6**

4. a) State and prove the parallel axis theorem. **7**

4. b) Calculate the moment of inertial of a cylinder about an axis passing through its center. **7**

5. Consider the following collision problem as shown in Fig. 03. **14**

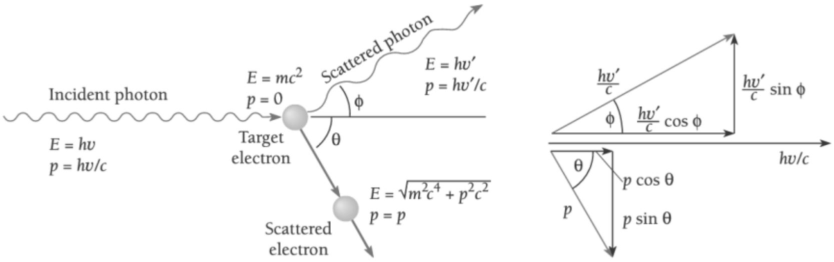

Fig. 03

By means of the momentum conservation, calculate the shift in wavelength $\lambda - \lambda'$ of the photon in the collision process.

6. Consider the motion of a harmonic oscillator with damping driven by a sinusoidal excitation. **14**

i. Model the equation of motion with differential equation
ii. Use complex number to propose solution ansatz for the differential equation
iii. Find out the expression of the impedance of the system using the method of complex exponential.

7. a) State and prove Bernoulli's theorem for fluid motion and give two of its applications. **7**

7. b) Define surface tension. Find the relation between the radius of (i) a spherical drop, (ii) a spherical bubble of a liquid, the surface tension, and pressure. **7**

8. a) Define center of mass for a two-body system. **4**

8. b) What is the advantage of working in a center of mass system for multiple object systems? **3**

8. c) Find out the center of mass for the object shown in Fig. 04. **7**

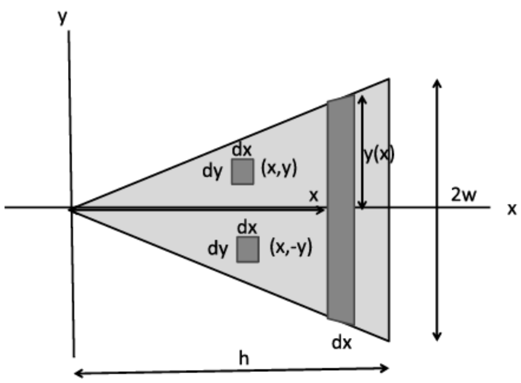

Fig. 04