Kishoreganj University

1^{st} Year 1^{st} Semester B.Sc. (Hons.) Final Examination 2024
Department of Mathematics
MAT 1101: Fundamentals of Mathematics (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Write down the definitions of set, cardinal number, and equivalent set with example. Using Vann diagrams determine whether $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ for all sets, A, B, and C. [6]

   (b) Liberty Travel surveyed 125 potential customers. The following information was obtained. 68 wished to travel to Hawaii. 53 wished to travel to Las Vegas. 47 wished to travel to Disney World. 34 wished to travel to Hawaii and Las Vegas. 26 wished to travel to Las Vegas and Disney World. 23 wished to travel to Hawaii and Disney World. 18 wished to travel to all three destinations. Use a Venn diagram to answer the following questions. How many of those surveyed a) did not wish to travel to any of these destinations? b) wished to travel only to Hawaii? c) wished to travel to Disney World and Las Vegas, but not to Hawaii? d) wished to travel to Disney World or Las Vegas, but not to Hawaii? e) wished to travel to exactly one of these destinations? [8]

2. (a) Describe conditional and biconditional statements with a truth table. Prove that $[p \Rightarrow (q \wedge r)]$ and $(p \Rightarrow q) \wedge (p \Rightarrow r)$ are logically equivalent. [6]

   (b) Use a truth table to determine whether the following argument is valid or invalid. [4]
   If my cell phone company is Verizon, then I can call you free of charge.
   I can call you free of charge or I can send you a text message.
   I can send you a text message or my cell phone company is Verizon.
   ___
   ∴ My cell phone company is Verizon.

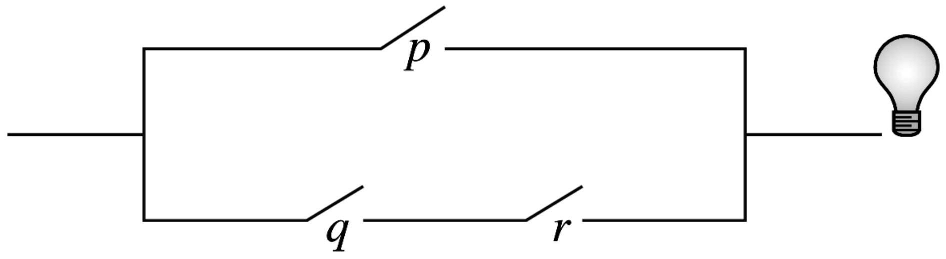
   (c) Draw the switching circuit that represent (i) $[(\boldsymbol{p} \vee q) \wedge r] \vee (\sim \boldsymbol{p} \wedge q)$ and (ii) $[(p \wedge \sim q) \vee (r \vee q)] \wedge s$. [4]

3. (a) Let the statements [6]
   p: Dinner includes soup.
   q: Dinner includes salad.
   r: Dinner includes the vegetable of the day.
   Write the following statements in symbolic form and construct truth table.

   i. Dinner includes soup, and salad or the vegetable of the day.
   ii. Dinner includes soup and salad, or the vegetable of the day.

   (b) What is implication? determine whether the statement $\neg p \to p$ is an implication. [4]

   (c) Write a symbolic statement that represents the circuit and construct a truth table to determine when the lightbulb will be on. [4]



4. (a) If z_1 and z_2 are two complex numbers then prove that $|z_1 + z_2| \leq |z_1| + |z_2|$. [4]
   (b) Represent graphically the set of values of z for which $|z + 3| + |z - 3| = 10$. [5]
   (c) Solve the equation $x^5 + x^4 + x^3 + x^2 + x + 1 = 0$ using De Moivre's theorem. [5]

5. (a) As a reward for saving his kingdom from a band of thieves, a king offered a knight one of two options. The knight's first option was to be paid 100,000 pounds of silver all at once. The second option was to be paid over the course of a month. On the first day, he would receive one pound of silver. On the second day, he would receive two pounds of silver. On the third day, he would receive four pounds of silver, and so on, each day receiving double the amount given on the previous day. Assuming the month has 30 days, which option would provide the knight with more silver? [5]
   (b) Sum the series [9]
      i. 1.3.5 + 2.4.6 + 3.5.7 + ... to n terms
      ii. $\frac{4}{1.2.3} + \frac{5}{2.3.4} + \frac{6}{3.4.5} + ...$ to n terms

6. (a) If there are n positive quantities $a_1, a_2, \ldots, a_n$ then prove that [5]

$$\frac{a_1^m + a_2^m + \cdots + a_n^m}{n} > \left(\frac{a_1 + a_2 + \cdots + a_n}{n}\right)^m$$

   except when m lies between 0 and 1.
   (b) Show that [5]

$$\frac{1}{\sqrt{2n+1}} > \frac{1.3.5\ldots\ldots(2n-1)}{2.4.6\ldots\ldots2n} > \frac{\sqrt{n+1}}{2n} \text{ or, } \frac{2}{2\sqrt{n+1}}$$

   (c) If a, b, and c are all positive quantities prove that [4]

$$\frac{a^4+b^4}{a^2+b^2} + \frac{b^4+c^4}{b^2+c^2} + \frac{c^4+a^4}{c^2+a^2} \geqslant a^2 + b^2 + c^2$$

   .

7. (a) Prove that every equation of degree n has exactly n roots. [4]
   (b) If a, b, c are the roots of the equation $x^3 + qx + r = 0$ form the equation whose roots are $\frac{1}{a} + \frac{1}{b}, \frac{1}{b} + \frac{1}{c}, \frac{1}{c} + \frac{1}{a}$. [4]
   (c) Transform the equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$ into one in which the term x^2 is absent. [6]

8. (a) Use the Borda count method and Plurality method to determine the winner of the election for president of the Honor Society where the candidates are Antoine (A), Betty (B), Camille (C), and Don (D). The preference table is shown bellow. [6+2]

Number of Votes	19	15	11	7	2
First	B	C	D	A	C
Second	A	A	C	D	D
Third	C	D	A	C	A
Fourth	D	B	B	B	B

(b) The Transit Department in the city of Houston has 100 new buses to be apportioned among six routes. The department decides to apportion the buses based on the average number of daily passengers per route, as shown in the table below. [6]

Route	A	B	C	D	E	F	Total
Passengers	9070	15,275	12,810	5720	25,250	6875	75,000

i Determine the standard divisor.

ii Determine each route's standard quota.

iii Determine each route's apportionment using Hamilton's method.

Kishoreganj University

1^{st} Year 1^{st} Semester B.Sc. (Hons.) Final Examination 2024
Department of Mathematics
MAT 1103: Differential Calculus (3 Credits)

Time: 03:00 Hours | **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define domain and range of a function. [2]

   (b) Determine the domain and range and also sketch the following function [9]
   (i) $y = \dfrac{x+1}{x-1}$, (ii) $f(x) = 2^{-x} - 3$, (iii) $f(x) = [x]$.

   (c) Show that if $f(x) = \sqrt{x}, g(x) = \sqrt{x}$, and $h(x) = x$, then the domain of fg is not the same as the natural domain of h. [3]

2. (a) A real function $f(x)$ is defined as follows: $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & ;\text{ when } x \neq 2 \\ 3 & ;\text{ when } x = 2 \end{cases}$ [6]

   Show that $f(x)$ is discontinuous at $x = 2$. Also, redefine $f(x)$ in such a way so that the function $f(x)$ is continuous at $x = 2$.

   (b) Discuss the continuity of the function $f(x) = \dfrac{1}{5 + e^{\frac{1}{x-2}}}$ at $x = 2$. [6]

   (c) Justify the function $f(x) = x^2 + 1$ where $x \in [1, 3]$ for the Intermediate Value Theorem. [2]

3. (a) Show that the limit $\lim\limits_{x \to 0} \dfrac{x}{|x| + x^2}$ does not exist. [2]

   (b) Prove that [5]

   (i) $\lim\limits_{x \to \infty} \dfrac{x^n}{e^x} = 0$

   (ii) $\lim\limits_{x \to 0} (1+x)^{1/x} = e$

   (c) Find the value of $\lim\limits_{x \to 0} \left\{ \dfrac{e^x - e^{\sin x}}{x - \sin x} \right\}$ [3]

   (d) Define Rolle's theorem. Verify this theorem for the function $f(x) = \cos 2x + 2\cos x$ in the interval $[0, 2\pi]$. [4]

4. (a) If $F(x) = f\left(xf\left(xf(x)\right)\right)$, where $f(1) = 2, f(2) = 3, f'(1) = 4, f'(2) = 5,$ and $f'(3) = 6.$ Find the value of $F'(1).$ [2]

   (b) Find the $\dfrac{dy}{dx}$ of the following function. [3*3=9]

   (i) $y = \sin\left(2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right)$

   (ii) $y = (x^2+1)^{\sqrt{1-x^2}} + f(x)^{\cos^{-1} x}$

   (iii) $y = \cos^n\left(\dfrac{ax}{y}\right)$

   (c) Find the differentiation of $x^{\sin x}$ with respect to $(\sin x)^x$. [3]

5. (a) Find the n^{th} derivative of the function $y = \ln(ax + b).$ [2]

   (b) State Leibniz theorem. Using Leibniz theorem to proved that $\dfrac{d^{n+1}}{dx^{n+1}}(x^n \ln x) = \dfrac{n!}{x}.$ [3]

(c) If $y = \sin(m \sin^{-1} x)$ then prove that $(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2-m^2)y_n$. Also find $y_n(0)$. [6]

(d) Use an appropriate local linear approximation to estimate the value of $\sqrt[3]{27.01}$. [3]

6. (a) Define critical point and inflection point of a function. Find the critical numbers and inflection point for the functions $f(x) = x^{2/3}(6-x)^{1/3}$. [5]

(b) Find the local extremum of the function $f(x) = x^4$ by 1^{st} derivative test. [3]

(c) Find the interval where the function $f(x) = x^3 - 3x^2 + 2x - 4$ is concave down and concave up also find the inflection point if exists. [6]

7. (a) A cylindrical biscuit tin has close-fitting lid which overlaps the tin by 1 cm . The radii of the tin and the lid are both x **cm**. The tin and the lid are made from a thin sheet of metal of 80π $\mathbf{cm^2}$ and there is no wastage. The volume of the tin is $\mathbf{Vcm^3}$. [6]

(i) Show that $V = \pi\left(40x - x^2 - x^3\right)$

(ii) Prove that there is maximum value of V and finds it.

(b) An open box is to be made from a **16**-inch by **30**-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume? [5]

(c) Find a point on the curve $y = x^2$ that is closest to the point$(18, 0)$. [3]

8. (a) Expand $e^{ax}\cos bx$ in finite Maclaurin's series with the remainder in Lagrange's form. [6]

(b) Use the root test to determine whether the following series converge or diverge [4]

(i) $\sum_{k=1}^{\infty}\left(\frac{3k+2}{2k-1}\right)^k$.

(ii) $\sum_{k=1}^{\infty}\frac{1}{\{\ln(k+1)\}^k}$.

(c) Use Taylor's theorem to expand $f(x) = \dfrac{1}{x+1}$ in power of $(x-3)$. [4]

Kishoreganj University

1^{st} Year 1^{st} Semester B.Sc. (Hons.) Final Examination 2024
Department of Mathematics
MAT 1105: Analytic Geometry (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Find the condition that the general equation of the second degree $bx^2 + 2hxy + ay^2 + 2gx + 2fy + c = 0$ may represent a pair of straight lines. [6]

   (b) Show that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel straight lines if $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$. Also show that the distance between them is $2\sqrt{\frac{g^2 - ac}{a(a+b)}}$. [8]

2. (a) Define the chord of contact. Show that the straight lines $y = mx \pm a\sqrt{1+m^2}$ are always tangents to the circles $x^2 + y^2 = a^2$. [4]

   (b) Find the equation of the circle whose diameter is the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$. [4]

   (c) Define limiting point and point circle. Find the coordinates of the limiting point of the circles $x^2 + y^2 + 2x + 4y + 7 = 0$ and $x^2 + y^2 + 4x + 2y + 5 = 0$. [6]

3. Reduce the equation of the conic $x^2 - 6xy + 9y^2 - 2x - 3y + 1 = 0$ to its standard form. Also find the axis of the conic, length of the latus rectum, coordinates of the vertex, coordinates of the foci, equation of the directrix. And sketch the rough graph of the conic. [14]

4. (a) If three normals from a point of the parabola $y^2 = 4ax$ cut the axis in points whose distance from the vertex is arithmetical progression, show that the point lies on the curve $27ay^2 = 2(x - 2a)^3$. [6]

   (b) The tangents at the extremities of a normal chord of parabola $y^2 = 4ax$ meet in a point T. Show that the locus of T is $(x + 2a)y^2 + 4a^3 = 0$. [4]

   (c) Define asymptotes. If e and e' be the eccentricities of a hyperbola and its conjugate hyperbola, then prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$. [4]

5. (a) Define direction cosine and direction ratio. Find the direction cosine of the line which is perpendicular to the lines with direction cosines proportional to $(1, -2, -2)$ and $(0, 2, 1)$. [4]

   (b) If P, Q, R, S are the points $(3, 4, 5)$, $(4, 6, 3)$, $(-1, 2, 4)$ and $(1, 0, 5)$, find the projection of RS on PQ. [4]

   (c) Prove that two lines whose direction cosines are given by the relations $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$ are perpendicular if $u(b^2+c^2) + v(c^2+a^2) + w(a^2+b^2) = 0$ and parallel if $\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$. [6]

6. (a) Define plane. Find the equation of the plane which passes through the point $(0, 4, -3)$ and $(6, -4, 3)$, and the sum of the intercepts made by the plane on the three axes be 0. [6]

   (b) Show that the distance between the two parallel planes $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z + 5 = 0$ is $\frac{1}{6}$. [4]

   (c) find the locus of the point such that the sum of the squares of its distances from the planes $x + y + z = 0, x - z = 0,$ and $x - 2y + z = 0$ is 9. [4]

7. (a) Find the equation of tangent planes to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which are parallel to the plane $lx + my + nz = 0$. [7]

(b) Find the shortest distance and equation of shortest distance line between the two lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$. [7]

8. (a) Define great circle. Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z-8=0$ is a great circle. [5]

(b) A sphere of constant radius r passes through the origin O and cuts the axes in A, B, C. Prove that the locus of the foot of the perpendicular from O to the plane ABC is given by $(x^2+y^2+z^2)^2(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2})=4r^2$. [4]

(c) Define the right circular cone. Find the equation of the right circular cone with vertex at $(2,1,-3)$ and whose axis is parallel to OY, and semi-vertical angle is 45^0. [5]

Kishoreganj University

1^{st} Year 1^{st} Semester B.Sc. (Hons.) Final Examination 2024
Department of Mathematics
STA 1107: Basic Statistics (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Distinguish (any two) between (i) grouped and ungrouped data, (ii) quantitative and qualitative variables, and (iii) exclusive and inclusive intervals. [4]

   (b) Weekly pocket expenses (in \$) of **35** students of class $\boldsymbol{XIV}$ are **37, 41, 39, 34, 71, 26, 56, 61, 58, 79, 83, 72, 64, 39, 75, 39, 37, 59, 57, 37, 53, 38, 49, 45, 70, 82, 44, 37, 79, 76, 44, 67, 81, 28, 79**. Construct a frequency distribution table (Inclusive Method). [6]

   (c) Compare two companies having the following data for **50** shares: [4]

	Company A	Company B
Average price of share (Tk.)	150	120
Standard deviation	15	8

   Which companies share you will buy? Explain.

2. (a) The expenditure (Taka in thousand) of families is given as below: [8]

Expenditure :	4-6	6-8	8-10	10-12	12-14
No. of families:	5	8	10	7	3

   Draw the histogram and ogive curve and then calculate the median from each of them.

   (b) A factory produces lamps. In an experiment on the working life of these lamps in the following: [6]

Lengths of life (Hours)	10-12	12-14	14-16	16-18	18-20	20-22	22-24
Number of lamp	13	33	42	62	40	10	8

   Calculate IQR, $\boldsymbol{D_4}$, and $\boldsymbol{P_{65}}$.

3. (a) From a certain frequency distribution consisting of 18 observations, the mean and standard deviation were computed to be 7 and 4 respectively. But on comparing the original data, it was found that observation 12 was misreported as 21 in the computation. Compute the correct mean and correct standard deviation. [4]

   (b) Show that for any set of values $\boldsymbol{x_1, x_2, \ldots, x_n}$, the mean deviation about the arithmetic mean cannot exceed the standard deviation. [4]

   (c) The first three moments of a distribution about the value 2 of a variable $\boldsymbol{x}$ are 1, 16, and -40. Find the mean, the variance, and the third central moment. Also show that the first three moments about $\boldsymbol{x = 0}$ are 3, 24, and 76. [6]

4. (a) Derive the general formula for central moments in terms of raw moments. [5]

   (b) The ages of the BPC workers arranged in a grouped frequency distribution are as follows. [9]

Age (in years)	24.5-29.5	29.5-34.5	34.5-39.5	39.5-44.5	44.5-49.5
Workers	3	9	15	12	7

   (i) Calculate the central moments.
   (ii) And then estimate the flatness and peakness of the data.

5. (a) The measure of skewness of a distribution is 0.3. The mode and the median are 50 and 55. Find the mean and standard deviation of the distribution. [4]

(b) The accompanying table shows the proportions of coal miners who exhibit symptoms of pneumoconiosis to their number of years of working in coal mines. [10]

Years	5	10	15	20	25	30	35	40	45	50
Proportions	0	.01	.02	.07	.15	.17	.18	.21	.35	.45

i) Calculate the regression line of proportion with pneumoconiosis (y) on working years $(\boldsymbol{x})$.
ii) Obtain the standard error of the estimate and hence find the coefficient of correlation between $\boldsymbol{x}$ and $\boldsymbol{y}$.
iii) Calculate the correlation coefficient directly from the formula and compare the same with the one obtained in (ii).
iv) Use your fitted regression line to estimate the proportion of coal miners developing pneumoconiosis who have worked for 42 years and 52 years.

6. (a) What is multiple regression? Discuss its uses and limitations. [4]

(b) A test was made to different doses of nitrogen $(\boldsymbol{N})$ on rice yild. The doses of phosphorus $(\boldsymbol{P})$ and potassium $(\boldsymbol{K})$ were kept at the optimum level for proper growth of the plant. The following data were recorded. [10]

Nitrogen doses (lb/acre)	0	1	2	3	4	5
Yield (md/acre)	15	25	40	55	52	43

Fit a second-degree polynomial to the data and obtain $\boldsymbol{R^2}$.

7. (a) What is Regression? What are the parameters of a simple regression model? How do you estimate these parameters? Estimate. [7]

(b) A nutrition study recorded a sample of **7** children under **5** years of age-weighted and their monthly family incomes in ‘**000** taka. The result is shown in the accompanying table. [7]

Family income (in ‘**000** taka):$\boldsymbol{x}$	13	20	34	24	16	30	36
Weight in kg:$\boldsymbol{y}$	15	19	21	16	12	16	18
Estimated weight: $\hat{\boldsymbol{y}}$	15.2	16.4	19.0	17.2	15.7	18.2	19.3

Calculate the coefficient of determination, MSE, and standard error, and interpret your results.

8. (a) Write the advantages of rank correlation coefficient over simple correlation coefficient. [4]

(b) A department store has the following statistics on sales $(\boldsymbol{y})$ for a period of last year of **7** salesmen, who have varying sales experience $(\boldsymbol{x})$ [10]

Years of experience	1	3	4	4	6	8	10
Annual sales (in ‘**000** taka)	80	97	92	102	103	111	119

(i) Estimate the regression line of annual sales volume $(\boldsymbol{y})$ on years of experience $(\boldsymbol{x})$.
(ii) Explain the intercept value and slope value.
(iii) Predict the annual sales volume of a person who has **20** years of sales experience. Also interpret your results.

Kishoreganj University

1st Year 1st Semester B.Sc. (Hons.) Final Examination 2024

Department of Mathematics

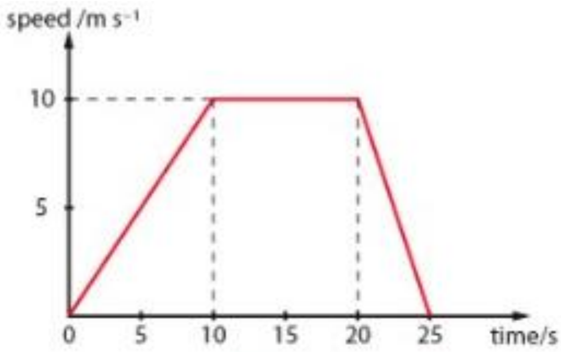
PHY 1109: Mechanics, Waves, and Properties of Matters (3 Credits)

Time: 3:00 Hours **Full Marks: 70**

Figures shown in the right margin indicate full marks.
Answer any 05 (Five) out of 08 (Eight) questions.

1 **a.** Discuss the difference between displacement and distance with an example. **1+4**

The figure shows the speed-time graph of a moving lift.



(i) What is the maximum speed of the lift?
(ii) For how many seconds does the lift move?
(iii) How much speed does the lift gain in the first 10 seconds? What is its acceleration?
(iv) What is the deceleration of the lift in the last 5 seconds?

b. Derive the kinematic equation of motion. **4**

c. A student flips a coin into the air. Its initial velocity is 8.0 ms^{-1}. Taking g = 10 ms^{-2} and ignoring air resistance, calculate (i) the maximum height, h, the coin reaches, (ii) the velocity of the coin on returning to his hand, and (iii) the time that the coin is in the air. **5**

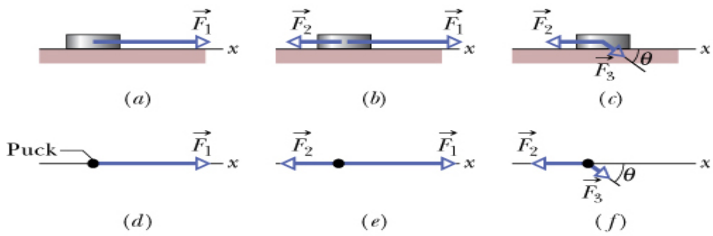
2 **a.** Define torque and moment of inertia. **3**

b. Show that the trajectory of a projectile is parabolic. Hence find out the maximum horizontal range of that projectile. **7**

c. A soccer player kicks a ball at an angle of 36° from the horizontal with an initial speed of 15.5 m/s. Assuming that the ball moves in a vertical plane, find (i) the time at which the ball reaches the highest point of its trajectory and (ii) its velocity when it strikes the ground. **4**

3 **a.** Derive the conservation of linear momentum using Newton's 2nd law of motion. **4**

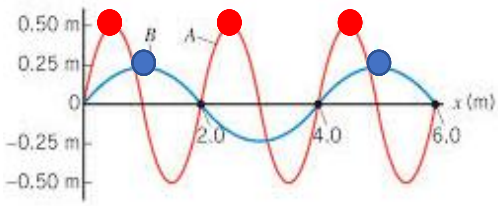
b. One or two forces act on a puck that moves over frictionless ice along an x-axis in one-dimensional motion. The puck's mass is m = 0.20 kg. Forces F_1 and F_2 are directed along the x-axis and have magnitudes F_1 = 4.0 N and F_2 = 2.0 n. Force F_3 is directed at angle q = 30° and has magnitude F_3 = 1.0 n. In each situation, what is the acceleration of the puck? **3×2 = 6**



c. A bullet of mass 65 grams is fired from a 4.0 kg gun. If the bullet leaves the gun at 800 m/s, what is the recoil velocity of the gun? **4**

4 **a.** (i) What is the wavelength of the red and blue waves? **4**

(ii) What is the amplitude of the red wave and blue wave?



(iii) If this is a snapshot of how far each wave traveled in 1 second, what is the frequency of the red wave? If this is a snapshot of how far each wave traveled in 1 second, what is the frequency of the blue wave?

(iv) Are the red and blue waves in phase with each other?

b. Consider a particle undergoing simple harmonic motion. The velocity of the particle at position x_1 is v_1 and the velocity of the particle at position x_2 is v_2. Show that the ratio of period and amplitude is, **4**

$$\frac{T}{A} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 x_2^2 - v_2^2 x_1^2}}$$

c. Derive the differential equation of linear simple harmonic motion. Does $\boldsymbol{x = Ae^{i\omega t}}$ represent simple harmonic motion? **4+2**

5 **a.** Explain what compression and rarefaction mean when applied to a sound wave traveling in the air. Describe the displacement and pressure variation of a sound wave from its equilibrium position. **5**

b. Obtain an expression for the beat frequency. **5**

c. A sound of 1500 Hz is emitted by a source that moves away from an observer and towards a cliff at a speed of 6 ms^{-1}. **4**

(i) Calculate the frequency of the sound which is coming directly from the source.

(ii) Compute the frequency of sound heard by the observer reflected off the cliff. Assume the speed of sound in air is 330 ms^{-1}

6 **a.** State and explain Hooke's law with the help of a stress-strain diagram. **4**

b. Show that the depression at the loaded end of a light cantilever is proportional to the cube of its length. **6**

c. A single cantilever of length 18 cm, breadth 2.6 cm, and thickness 1 mm is loaded with 50 g at the free end. Find the depression produced at the free end of the cantilever. (Given that Young's modulus of the given material is $21 \text{x} 10^{10}$ Nm^{-2}.) **4**

7 **a.** Prove that the surface tension of a liquid is equal to the mechanical part of its surface energy. **5**

b. Show that the excess pressure inside a soap bubble of radius r over the atmospheric pressure outside it is equal to ***4T/r***, where ***T*** is the surface tension of the soap solution. **5**

c. A sphere of water, of radius 1 mm, is sprayed into a million drops of equal size. Find the work expended in doing so. **4**

8 **a.** Distinguish between streamline and turbulent flow of a liquid. **3**

b. Derive Bernoulli's equation for a fluid in streamline motion. **8**

c. A plate of metal 10^{-2} m^2 area rests on a layer of castor oil 2×10^{-3} m thick whose coefficient of viscosity is 1055 Ns/m^2. Calculate the horizontal force required to move the plate with a uniform speed of 3×10^{-2} m/s. **3**