Kishoreganj University

Department of Mathematics

2^{nd} Year 1^{st} Semester B.Sc. (Honours) Final Examination-2024

Course Code: MAT 2101 Course Title: Real Analysis

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) State Dedekind's theorem. Write down the Completeness axiom for the system of real numbers. [4]

   (b) Show that, the Completeness axiom is equivalent to Dedekind's theorem. [7]

   (c) Define supremum and infimum of a set. Find the supremum and infimum of the set: $S = \{x \in \mathbb{R} : 6x^2 - x - 1 < 0\}$. [3]

2. (a) What is neighbourhood of a point? What is open set, explain with an example. Give an example of (i) An open set which is not an interval. (ii) An interval which is an open set. (iii) A set which is neither an interval nor an open set. [5]

   (b) Prove that a sequence $\{a_n\}$ is bounded if and only if $\exists$ a positive real number M such that $|a_n| \leq M \ \forall n \in \mathbf{N}$. [4]

   (c) What is limit of a sequence? Prove that every monotonically increasing sequence which is bounded above converge to its least upper bound. [5]

3. (a) By using the definition show that the sequence $\{\frac{1}{3^n}\}$ converges to 0. [4]

   (b) Show that every Cauchy sequence is bounded. [4]

   (c) Prove, by definition, that the sequences whose n^{th} terms are given below are Cauchy sequences:(i) $\frac{1}{n^2}$ (ii) $\frac{n}{n+1}$. [6]

4. (a) Prove that if $\sum u_n$ and $\sum v_n$ converge to u and v respectively, then $\sum(u_n - v_n)$ converges to (u - v). [4]

   (b) Examine the convergence of the series $\sum(\sqrt{n^4+1} - n^2)$. [5]

   (c) Show that, for every $a > 0$, the sequence $a, \sqrt{a}, \sqrt[3]{a}, \sqrt[4]{a}, ...$ converges to 1. [5]

5. (a) If $f(x) \leq g(x) \leq h(x)$ and $\lim_{x\to a} f(x) = l = \lim_{x\to a} h(x)$, then prove that $\lim_{x\to a} g(x)$ exists and is equal to l. [4]

   (b) Discuss the continuity of the function f(x)=[x] at the points $\frac{1}{2}$ and 1, where [x] denotes the largest integer $\leq$ x. [6]

   (c) Using Intermediate value theorem, show that $x^{41} + x + 1 = 0$ has a real root. [4]

6. (a) State Rolle's theorem. Show that the function $f(x) = 1 - (x-1)^{\frac{2}{3}}$ satisfies $f(0) = f(2)$, but there is no point c in $(0, 2)$ such that $f'(c) = 0$. Why is this not a violation of Rolle's theorem? [4]

   (b) When a function is said to be Darboux integrable on a closed interval. Prove that, a bounded function f is Darboux integrable on [a,b] iff for each $\epsilon > 0$ there exists a partition P of $[a, b]$ such that $U(f, P) - L(f, P) < \epsilon$. [5]

(c) Show that, a constant function is Riemann integrable. [5]

7. (a) Define metric space with an example. Let (M, d) be a metric space. Construct a function $\rho(x, y) = \frac{d(x,y)}{1+d(x,y)}$. Then show that (M, ρ) is a metric space and $\rho(x, y)$ is bounded by 1. [4]

   (b) Prove that neighbourhood of a point in a metric space is an open set. [4]

   (c) Define continuous function on a metric space. Prove that a mapping $f : X \to Y$, where X and Y are metric space is continuous if and only if $f^{-1}(V)$ is open in X for every open set V in Y. [6]

8. (a) What is Cauchy sequence? Prove that every convergent sequence sequence is a Cauchy Sequence. [7]

   (b) What is complete metric space? Let $\mathbf{X} = C[0, 1]$ denote the class of continuous functions on $[0, 1]$. The metric d is defined by $d(f, g) = \max_{0 \le x \le 1} |f(x) - g(x)| \; \forall f, g \in \mathbf{X}$. Show that $(\mathbf{X}, d)$ is a complete metric space. [7]

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Department of Mathematics

2^{nd} Year 1^{st} Semester B.Sc. (Honours) Final Examination-2024

Course Code: MAT 2103 Course Title: Differential Calculus of Several Variables

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define vector valued function. Describe the tangent lines of vector-valued function. Find the domain of the function $\mathbf{F}(t) = \mathbf{i}\sin \pi t + \mathbf{j}\dfrac{t^2}{t+1} + \mathbf{k}\sqrt{2+t}$. Find $\mathbf{F}(0)$. Is the functional value exists at $t = -3$? [8]

   (b) Sketch the circle $\mathbf{r} = \mathbf{i}\cos t + \mathbf{j}\sin t$ and in each part draw the following vector with its correct length [4]

   (i) $\mathbf{r}''(\pi)$.

   (ii) $\mathbf{r}(2\pi) - \mathbf{r}\left(\dfrac{3\pi}{2}\right)$.

   (c) Find the intersection point of the straight line $\mathbf{r} = t\mathbf{i} + (2t+1)\mathbf{j} + (t-1)\mathbf{k}$ and the plane $2x - y + z = 0$. [2]

2. (a) Find the arc length parameterization of the line $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$ that has reference point $\mathbf{r}_0$ and the same orientation as the given line. [4]

   (b) Determine the arc-length parametrization of the line $x = 2t + 1,\ y = 3t - 2$. [4]

   (c) If the velocity of a particle moving in space is $\mathbf{r}'(t) = \mathbf{i}\cos t - \mathbf{j}\sin t + \mathbf{k}$ and the position vector is $\mathbf{r} = 2\mathbf{i} + \mathbf{j}$ at $t = 0$, then find the position vector of the particle at time t. [4]

   (d) If $\mathbf{r}(t) = \mathbf{i}\cos t + \mathbf{j}\sin t + \mathbf{k}$, then determine [2]

   (i) $\lim_{t\to 0} (\mathbf{r}(t) \cdot \mathbf{r}'(t))$

   (ii) $\lim_{t\to 0} (\mathbf{r}(t) \times \mathbf{r}'(t))$

3. (a) Find the center of curvature and radius of curvature of the curve $x^2 + 4y^2 = 25$ at the point $(3, 2)$. [4]

   (b) Find the chord of curvature through the pole of the curve $r^n = a^n \cos n\theta$. [4]

   (c) Define limit and continuity of a function of two variables. Let [6]

$$f(x,y) = \begin{cases} -\dfrac{xy}{x^2+y^2}, & \text{when} \quad (x,y) \neq (0,0) \\ 0, & \text{when} \quad (x,y) = (0,0) \end{cases}.$$

   (i) Show that $f_x(x,y)$ and $f_y(x,y)$ exist at all points (x, y),
   (ii) Explain why f is not continuous at $(0,0)$.

4. (a) Prove that by using (δ, ε), $\lim_{(x,y)\to(-2,3)} (2x^2 - 3y) = -1$. [3]

(b) How are the partial derivatives f_x and f_y of a function $f(x,y)$ are defined? How are they physically interpreted? Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$ for [6]

$$f(x,y) = \begin{cases} \dfrac{xy\,(x^2-y^2)}{(x^2+y^2)} & ;(x,y) \neq (0,0) \\ 0 & ;(x,y) = (0,0) \end{cases}.$$

(c) What does it mean for a function $f(x,y)$ to be differentiable? What is the chain rule? What form does it take for functions of two and three independent variables? How do you diagram these different forms? Let $\omega = e^{xyz}, x = 3r+s, y = 3r-s, z = r^2 s$. Use appropriate forms of the chain rule to find $\frac{\partial \omega}{\partial r}$ and $\frac{\partial \omega}{\partial s}$. [5]

5. (a) Take into account $u = \ln(x^3+y^3+z^3-3xyz)$, then show that [6]

(i) $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = \dfrac{3}{x+y+z}$

(ii) $\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z}\right)^2 = \dfrac{-9}{(x+y+z)^2}$.

(b) If $u = f(x,y)$ where x and y are dependent or independent variables then show that $du = \dfrac{\partial u}{\partial x}dx + \dfrac{\partial u}{\partial y}dy$. [4]

(c) Suppose $v = v(x,y,z)$ and $f(v^2-x^2, v^2-y^2, v^2-z^2) = 0$, then prove that [4]

$$\frac{1}{x}\frac{\partial v}{\partial x} + \frac{1}{y}\frac{\partial v}{\partial y} + \frac{1}{z}\frac{\partial v}{\partial z} = \frac{1}{v}.$$

6. (a) State and prove Taylor's series. Find Taylor's finite series with Lagrange's form of the remainder of the function e^{x+y} in the neighborhood of $(0,0)$. [6]

(b) Establish the Lagrange's method of undetermined multipliers for maximum value and minimum value. [4]

(c) Find the relative maximum and minimum values of $f(x,y) = x^3+y^3-3x-12y+20$. [4]

7. (a) Define directional derivative, and also determine the maximum value of the directional derivative. Find the directional derivative of $f(x,y) = \tan^{-1}\left(\dfrac{y}{x}\right)$ at $P(-2,2)$ in the direction of $\mathbf{a} = -\mathbf{i} - \mathbf{j}$. [7]

(b) Define tangent plane and normal line. Find the equation of the tangent plane and normal line to the surface $x^2+y^2 = z$ at the point $(2,-1,5)$. [7]

8. (a) Define divergence of a vector and interpret its physical significance. Find the acute angle between the surfaces $xy^2z = 3x+z^2$ and $3x^2-y^2+2z = 1$ at the point $(1,-2,1)$. [7]

(b) Define curl of a vector. Find an equation for the tangent plane to the surface $xz^2+x^2y = z-1$ at the point $(1,-3,2)$. [3]

(c) Find the value of (i) $\nabla^2(\frac{1}{\mathbf{r}})$, and (ii) $\nabla^2(\mathbf{r}^n)$, where $\mathbf{r}$ is a position vector of any point and n is a constant. [4]

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2nd Year 1st Semester B.Sc. (Honours) Final Examination-2024

Course Code: MAT 2105 Course Title: Ordinary Differential Equations-I

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define with an example of the following: [6]
   (i) Linear and Nonlinear differential equation.
   (ii) Order and Degree of a differential equation.
   (iii) Initial value problem (IVP) and Boundary value problem (BVP).

   (b) Write down the order-degree of the following differential equation [3]
   (i) $\frac{d^6x}{dt^6} + \left(\frac{d^4x}{dt^4}\right)\left(\frac{d^3x}{dt^3}\right) + x = t$
   (ii) $\frac{d^2r}{d\theta^2} = \sqrt[3]{\frac{dr}{d\theta} + r}$
   (iii) $\left(\frac{d^2y}{dx^2}\right)^3 - 2\left(\frac{dy}{dx}\right)^4 + 3y = \sin x$

   (c) Find the differential equation of the family of parabola with vertex on $y-$axis, axis parallel to the $x-$axis and distance from the focus to vertex fixed as a. Sketch some representative members of the family. [5]

2. (a) Define Homogeneous DE and solve the initial value problem $(2x - 5y)dx + (4x - y)dy = 0, y(1) = 4$. [6]

   (b) Solve the differential equation $(1 + x)dy - dx = 0$. [4]

   (c) Find the general solution of $\frac{dy}{dx} + y = e^{3x}$ and also determine whether there are any transient terms in the general solution. [4]

3. (a) A certain radioactive substance has a half-life of 35 years. How long does it take for 70% of the radioactivity to be dissipated? [5]

   (b) The population $x(t)$ of a certain city satisfies the logistic law [9]

   $$\frac{dx}{dt} = \frac{x}{100} - \frac{x^2}{10^8}, \quad \text{where } t \text{ is measured in years.}$$

   (i) If population of the city is 100000 in 1980, find a formula for the population in the future years.
   (ii) What will be the population in 2000?
   (iii) In what year does 1980 the population double?
   (iv) How large the population ultimately be?

4. (a) If $y_1 = e^x$ is a solution of $y'' - y = 0$ on the interval $(-\infty, \infty)$, find a second solution y_2 by using the reduction of order. [4]

(b) Use the method of undetermined coefficients to solve the initial value problem $y'' + y = 4x + 10\sin x,\ y(\pi) = 0,\ y'(\pi) = 2.$ [6]

(c) State the superposition principle for Homogeneous Equations. Also, determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$, $f_1(x) = x, f_2(x) = x^2, f_3(x) = 4x - 3x^2$. [4]

5. (a) Solve the differential equation $(D^4 - 4D^3 + 14D^2 - 20D + 25)y = 0$. [4]

(b) By the method of variation of parameter, solve $\dfrac{d^2y}{dx^2} + y = \tan x$. [6]

(c) A mass weighing 8 pounds stretches a spring 2 feet. Assuming that a damping force numerically equal to 2 times the instantaneous velocity acts on the system, determine the equation of motion if the mass is initially released from the equilibrium position with an upward velocity of $3ft/s$. [4]

6. (a) Using the method of variation of parameters, solve the following differential equation. [8]

(i) $y'' + 4y = 4\tan 2x$

(ii) $y'' + 4y' + 5y = e^{-2x}\sec x$

(b) Solve the following system of differential equations by systematic elimination [6]

$$Dx + (D + 2)y = 0$$
$$(D - 3)x - 2y = 0$$

7. (a) Define convergent and divergent series. Find the interval and radius of convergence for $\sum_{n=1}^{\infty} \dfrac{(x-3)^n}{2^n n}$. [6]

(b) State the Frobenius theorem.If $x = 0$ is a regular singular point of the differential equation $3xy'' + y' - y = 0$, then use the method of Frobenius to obtain the general solution. [8]

8. (a) Define solution vector. Verify that on the interval $(-\infty, \infty)$ $X_1 = \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix}$ and $X_2 = \begin{bmatrix} 3e^{6t} \\ 5e^{6t} \end{bmatrix}$ are solutions of $X' = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} X$. [3]

(b) Find the general solution of the given system [4]

$$\frac{dx}{dt} = -4x + y + z$$
$$\frac{dy}{dt} = x + 5y - z$$
$$\frac{dz}{dt} = y - 3z$$

(c) Solve the system $X' = \begin{bmatrix} 6 & 1 \\ 4 & 3 \end{bmatrix} X + \begin{bmatrix} 6t \\ -10t + 4 \end{bmatrix}$ on $(-\infty, \infty)$ by using undetermined coefficient method. [7]

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2^{nd} Year 1^{st} Semester B.Sc. (Honours) Final Examination-2024

Course Code: MAT 2107 Course Title: Mathematical Statistics

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define population, sample, parameter, and standard error with examples. [4]

   (b) Write short notes on the probability density function and the probability mass function. [4]

   (c) Let (X, Y) be a two-dimensional non-negative continuous random variable having the joint density [6]

$$f(x,y) = \begin{cases} 4xye^{-(x^2+y^2)}; & x \geq 0,\ y \geq 0 \\ 0; & \text{elsewhere} \end{cases}.$$

   Determine the density function of $U = \sqrt{X^2 + Y^2}$.

2. (a) Define a random variable. Give two real-life examples of a continuous random variable. [2]

   (b) Explain conditional expectation. For any random variables X and Y, prove that [6]

$$E[E(Y \mid X)] = E(Y)$$

   (c) The discrete random variables X and Y have the following probability function [6]

	X		
Y	1	2	Total
0	1/8	1/8	2/8
1	2/8	4/8	6/8
Total	3/8	5/8	1

   Find (i) $E(X \mid Y)$, (ii) $E(Y \mid X)$, and (iii) $V(X \mid Y = 0)$.

3. (a) The probability density function of the random variable X is as follows [5]

$$f(x) = \frac{1}{2}e^{-|x|}, \quad -\infty < x < \infty.$$

   Determine the characteristic function and hence the first three moments about the origin.

   (b) If $P(t)$ is the probability generating function for an integer-valued random variable X, then the rth factorial moment of X is given by [4]

$$\left.\frac{d^r}{dt^r}P(t)\right]_{t=1} = P^r(1) = \mu_{[r]}$$

   (c) State and prove Chebyshev's inequality. [5]

4. (a) Establish the relation between F and t-distribution. [6]

   (b) If X is a chi-square variate with n degrees of freedom, then for large n prove that $\sqrt{2X} \sim N(\sqrt{2n-1}, 1)$. [8]

5. (a) Show that the linear combination of k independent identically distributed normal variates is also a normal variate. [6]

   (b) The moment generating function of standard normal distribution is given by $e^{\frac{1}{2}t^2}$. Hence, find the coefficient of skewness (β_1) and comment on your findings. [4]

   (c) In a normal distribution with mean 25 and variance 9, what are the values that cover the central 50 percent of the area under the curve? [4]

6. (a) A coin is tossed 200 times. Find the approximate probability that the number of head is obtained between 80 to 120. [3]

   (b) Describe the process of testing equality of two population means when population variances are known and equal. [4]

   (c) The sample data show that 120 adult males born in rural area have a mean height of 62.7 inches with a standard deviation of 2.5 inches, and that 150 adult males born in urban area have a mean height of 61.8 inches with a standard deviation of 2.62 inches. Test the hypothesis (use 1% level of significance) that the mean heights in the two areas from which the samples have been drawn do not differ. Compute also the 95% confidence interval for the difference in population means. [7]

7. (a) Define simple hypothesis, composite hypothesis, critical region, and level of significance. [4]

   (b) Write down the test procedure of a single mean test. [5]

   (c) The gasoline mileage of 80 cars of make A has a mean of 20 miles/gallon with a standard deviation of 3 miles/gallon, while for 60 cars of make B the corresponding quantities are 22.5 and 5 (miles/gallon) respectively. Do the above data indicate any real difference in the distance covered per gallon if cars of the two makes? [5]

8. (a) Let we have a random sample of 21 values which yields an estimate of 8.5 for the variance of the population. Does this result support the hypothesis that population variance is 10? Find 95% confidence interval for the unknown parameter σ^2? [4]

   (b) Explain the term statistic, estimator, and estimate. [4]

   (c) What are the criteria of estimation? Show that the sample mean and sample variance are unbiased estimators of the population mean and population variance, respectively. [6]

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2^{nd} Year 1^{st} Semester B.Sc. (Honours) Final Examination-2024

Course Code: MAT 2109 Course Title: Introduction to Financial Mathematics

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) What is Compound Interest? State the formula for compound interest in t years with n compounding periods per year and annual interest rate r. Derive the formula for continuously compounded interest. [6]

   (b) Suppose an account earns 5.75% annually compounded monthly. If the principal amount is \$3104 then after three and one half years what will be the compound amount? What will be the effective interest rate? [4]

   (c) Suppose you wish to buy a house costing \$200000. You will put a down payment of 20% of the purchase price and borrow the rest from a bank for 40 years at a fixed interest rate r compounded monthly. If you wish your monthly mortgage payment to be \$1200 or less, what is the maximum annual interest rate for the mortgage loan? [4]

2. (a) Write the definition of the following terms: Dividends, Future contact, Zero coupon bond. [3]

   (b) What is the difference between a forward contract to buy an asset at \$30 and a call option to buy the same asset for \$30 ? [2]

   (c) Write down the definitions, payoff functions, profit functions and their graphs for the following term: Short Call, Long Put. [4]

   (d) A stock price is \$29. An investor buys one call option contract on the stock with a strike price of \$30 and sells a call option contract on the stock with a strike price of \$32.50. The market prices of the options are \$2.75 and \$1.50, respectively. The options have the same maturity date. Describe the investor's position. [5]

3. (a) What is the difference between an American and a European option? [2]

   (b) An investor buys a European call option with strike price of K and maturity T and sells a put option with the same strike price and same maturity. Describe the investor's position. [5]

   (c) Suppose that a European call option to buy a share for \$100.00 costs \$5.00 and is held until maturity. Under what circumstances will the holder of the option make a profit? Under what circumstances will the option be exercised? Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option. Find the value of the stock price S_T for [7]

      (i) The share will be exercised.

      (ii) The share will not be exercised.

      (iii) The investor make profit.

      (iv) The share will be exercised but the investor cannot make profit.

What is the investor's maximum gain and maximum loss? Draw a diagram illustrating your findings.

4. (a) What is a lower bound for the price of a 6-month call option on a non-dividend-paying stock when the stock price is \$80, the strike price is \$75, and the risk-free interest rate is 10% per annum? [4]

   (b) Suppose that c_1, c_2, and c_3 are the prices of European call options with strike prices K_1, K_2, and K_3, respectively, where $K_3 > K_2 > K_1$ and $K_3 - K_2 = K_2 - K_1$. All options have the same maturity. Show that $c_2 \leq 0.5\,(c_1 + c_3)$ [5]

   (c) Find a lower bound for the European call option with the exercise price \$15 when the stock price is \$21, the time to maturity is six months, and the risk-free interest rate is 8% p.a. Also show that there exists an arbitrage opportunity if we consider the European call option price is \$5. [5]

5. (a) What is brownian motion? Let $B(t)$ and $B^*(t)$ be two independent Brownian motions and assume $Z(t) = \rho B(t) + \sqrt{1-\rho^2}B^*(t),\ 0 \leq \rho \leq 1$. Verify that $Z(t)$ follows the properties of Brownian motion. Also, find $\text{Corr}[Z(t), B(t)]$. [8]

   (b) Using Itô's formula to show that for a function f of t and Brownian motion $B(t)$ the differential $df(t, B(t))$ is given by $df = \left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial B^2}\right) dt + \frac{\partial f}{\partial B} dB$. [6]

6. (a) Suppose that a stock price has an expected return of 36% per annum and a volatility of 40%. When the stock price at the end of a certain day is \$80, calculate the following [5]

      (i) the expected stock price at the end of the next day.

      (ii) the standard deviation of the stock price at the end of the next day.

   (b) Derive Itô's Lemma using Taylor Series. [5]

   (c) Show that the stochastic differential equation (SDE) for $f(S(t)) = \ln S(t)$ is [4]

$$\ln S(t) - \ln S_0 = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W.$$

7. (a) State the Black-Scholes formula for the call price $C(S,t)$ and put price $P(S,t)$. Then find the limit $\lim_{t\to 0} C(S,t)$ and $\lim_{t\to 0} P(S,t)$. [8]

   (b) What is Hedging? Calculate the price of a 3-month European put option on a non-dividend-paying stock with a strike price of \$50 when the current stock price is \$50, the risk-free interest rate is 10% per annum, and the volatility is 30% per annum. [6]

8. (a) What is a Bear Spread? Define the profit function for a Bear Spread and illustrate it on a graph. [5]

   (b) Options on a stock are available with strike prices of 65 BDT, 55 BDT, and 60 BDT, and expiration dates in 3 months. Their prices are 8 BDT, 3 BDT, and 5 BDT, respectively. Explain how the options can be used to create a butterfly spread. Construct a table showing how profit varies with stock price for the butterfly spread. For what range of stock prices would the butterfly spread lead to a loss? [5]

   (c) Calculate the payoff functions and then draw the payoff diagrams for the following portfolio: Long two calls and one put, all with strike price K. What is the benefit to hold such portfolios? [4]

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2^{nd} Year 1^{st} Semester B.Sc. (Honours) Final Examination-2024

Course Code: MAT 2111 Course Title: FORTRAN Programming

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Sketch a block diagram of a typical computer. Write down the history of the computer in detail. [4]

   (b) Distinguish between i) RAM and ROM, and ii) Machine language and High-level language. [5]

   (c) Convert $(29.325)_{10}$ to its binary equivalent and $(111001.01)_2$ to its decimal equivalent. [5]

2. (a) (i) Subtract 35 from 20 using complementary method. [6]
   (ii) Divide 53.7 by 2 using binary division method.
   (iii) Divide 69 by 10 using method of addition.

   (b) Define an algorithm and a flowchart. Describe any six standard symbols used in flowcharts as defined by ANSI. [8]

3. (a) Explain logical operators. If $a = 4, b = 2, c = 6, d = 5$, then find the value of the following expressions: [5]

$$Z = (a > b.AND.b > c.AND.d > a).\ \text{NEQV. .NOT.}\ (a > c.OR.b < a.AND.b < d)$$

   (b) Write short notes about character expression and implicit none statement. [4]

   (c) Write FORTRAN expression for the following Mathematical statements: [5]
   i. $\log|x-y| + e^{-x^2}$
   ii. $\left|\sqrt{x - y^3 - \frac{z^3}{\cos(a+b)}}\right|$
   iii. $\frac{\sqrt[3]{a^2}+\sqrt{a^3}}{\sqrt{a\sqrt{b}+a}}$

4. (a) Describe Nested Block If statement, ELSE and ELSE IF statement, and CASE statement. [3]

   (b) Write a FORTRAN program to evaluate $3 + 8 + 13 + \ldots\ldots + 38$. [4]

   (c) Write a flowchart and FORTRAN program to solve the quadratic equation [7]

$$Ax^2 + Bx + C = 0.$$

5. (a) Briefly explain STOP and END statement in FORTRAN. How do they differ from each other? [4]

   (b) Write the general form of Block IF statement and Arithmetic IF statement with their flowchart. [4]

(c) Write a program that reads a four-digit number and find the sum of the individual digits. Execute this program line by line for 4567. [6]

6. (a) Briefly explain parameter statement with an example. [2]

(b) What do you mean by main program and subprogram. Define a function subprogram and a subroutine subprogram and explain the difference between them. [4]

(c) If M is a $l \times m$ matrix and N is a $m \times n$ matrix, then write the FORTRAN program for the product matrix P of M and N, using subroutine. [8]

7. (a) Define an array and dimension statement in FORTRAN. Find the number of elements in the array: DIMENSION $L(1:10), M(-10:10), N(0:0)$. [4]

(b) Which of the following array names are valid or invalid and why? [4]
(*i*) ETA(5*2, 6) (*ii*) BETA(L, M, X) (*iii*) ALPHA(N+5, M) (*iv*) GAMMA(I=J*K).

(c) How can you input and output three-dimensional array elements using DO loops and implied DO loops in FORTRAN? Explain with an example for $I, J, K = 1, 2, 3$. [6]

8. (a) What is meant by format-directed input and output? Explain the $I-$format, $F-$format, and $E-$format specification statement in FORTRAN. [4]

(b) Given the following subprograms: [4]

```
FUNCTION SUM(U, V, W)
SUM=U+V+W
U=V+W
V=U+W
W=V+U
RETURN
END
```

```
SUBROUTINE ADD(U, V, W, SUM)
U=V+W
V=U+W
W=V+U
SUM=U+V+W
RETURN
END
```

Find the output of the following programs:

```
      X = 1.0
      Y = 2.0
      Z = 3.0
      TOTAL = SUM(X,Y,Z)
      CALL   ADD(X,Y,Z,GTOTAL)
      WRITE(6,800) X,Y,Z,TOTAL,GTOTAL
800   FORMAT(1X,5(F5.1,2X))
      END
```

(c) Explain the open file, close file and file input-output statements. [6]