Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 2nd Semester B.Sc (Hons.) Final Examination-2023
Department of Mathematics
MAT 1201: Linear Algebra I (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

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1. (a) Define Hermitian and shew-Hermitian matrices in $\mathbb{C}^n$. Show that iA is hermitian and A is a shew-hermitian matrix, where $A = \begin{pmatrix} i & 1+i & 2-3i \\ -1+i & 2i & 1 \\ -2-3i & -1 & 0 \end{pmatrix}$ [6]

   (b) Prove that, every square matrix can be expressed as $P + iQ$, where P and Q are hermitian matrices. [4]

   (c) Find all 2×2 matrices of the form $A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ such that A is involuntary matrix. [4]

2. (a) Solve by Gauss-Jordan elimination [5]

$$\begin{aligned} x_1 + 3x_2 - 2x_3 + 2x_5 &= 0, \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1, \\ 5x_3 + 10x_4 + 15x_6 &= 5, \\ 2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 &= 6. \end{aligned}$$

   (b) Define symmetric and skew-symmetric matrices with examples. Take into account $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 0 \end{pmatrix}$ and $B = B^T$. Is the matrix $(A+B)^3$ symmetric? [4]

   (c) Consider the matrices $A = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}$. [5]
   Compute Trace $\left(B^T A^T + 2C^T\right)$ (where possible).

3. (a) Determine the value of λ such that the following system of linear equation has (i) exactly one solution (ii) infinitely many solution and (iii) no solution. [5]

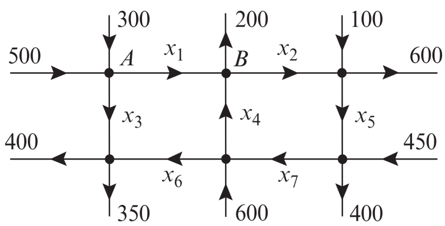
$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (\lambda^2 - 14)z &= \lambda + 2 \end{aligned}$$

   (b) The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour.

   (a) Set up a linear system whose solution provides the unknown flow rates. [2]

   (b) Solve the system for the unknown flow rates. [5]

   (c) Is it possible to close the road from A to B for construction and keep traffic flowing on the other streets? Explain. [2]



4. (a) What is vector space and subspace? List all the properties that the vector spaces are hold. [4]

(b) Let $\boldsymbol{V}$ be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $\underline{\boldsymbol{u}} = (\boldsymbol{u_1}, \boldsymbol{u_2})$ and $\underline{\boldsymbol{v}} = (\boldsymbol{v_1}, \boldsymbol{v_2})$: [7]

$$\underline{\boldsymbol{u}} + \underline{\boldsymbol{v}} = (\boldsymbol{u_1} + \boldsymbol{v_1} + \boldsymbol{1}, \boldsymbol{u_2} + \boldsymbol{v_2} + \boldsymbol{1}), \quad \boldsymbol{k}\underline{\boldsymbol{u}} = (\boldsymbol{k u_1}, \boldsymbol{k u_2})$$

i. Compute $\underline{\boldsymbol{u}} + \underline{\boldsymbol{v}}$ and $\boldsymbol{k}\underline{\boldsymbol{u}}$ for $\underline{\boldsymbol{u}} = (\boldsymbol{0}, \boldsymbol{4})$, $\underline{\boldsymbol{v}} = (\boldsymbol{1}, -\boldsymbol{3})$ and $\boldsymbol{k} = \boldsymbol{2}$.

ii. Show that $(\boldsymbol{0}, \boldsymbol{0}) \neq \underline{\boldsymbol{0}}$

iii. Show that $(-\boldsymbol{1}, -\boldsymbol{1}) = \underline{\boldsymbol{0}}$

(c) Determine whether the subset $\boldsymbol{W}$ is a subspace of $\mathbb{R}^3$ or not, where [3]

$$\boldsymbol{W} = \{(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}) \mid \boldsymbol{3a} - \boldsymbol{2b} + \boldsymbol{5c} = \boldsymbol{0}, \quad \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}\}$$

5. (a) Let $\boldsymbol{U}$ and $\boldsymbol{W}$ be the subspaces of $\mathbb{R}^4$ defined by [8]

$$\boldsymbol{U} = \{(\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{0}, -\boldsymbol{1}), (\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}, \boldsymbol{0}), (\boldsymbol{2}, \boldsymbol{3}, \boldsymbol{3}, -\boldsymbol{1})\}$$
$$\boldsymbol{W} = \{(\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{2}, -\boldsymbol{2}), (\boldsymbol{2}, \boldsymbol{3}, \boldsymbol{2}, -\boldsymbol{3}), (\boldsymbol{1}, \boldsymbol{3}, \boldsymbol{4}, -\boldsymbol{3})\}$$

Find a basis and the dimension of $\boldsymbol{U}, \boldsymbol{W}$ and $\boldsymbol{U} \cap \boldsymbol{W}$. respectively.

(b) Find the coordinate vector of $\underline{\boldsymbol{v}}$ related to the basis $\boldsymbol{S} = \{\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \underline{\boldsymbol{v}}_3\}$ for the vector space $\mathbb{R}^3$, where $\underline{\boldsymbol{v}} = (\boldsymbol{7}, \boldsymbol{4}, \boldsymbol{2})$, $\underline{\boldsymbol{v}}_1 = (\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{1})$, $\underline{\boldsymbol{v}}_2 = (-\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{2})$, and $\underline{\boldsymbol{v}}_3 = (\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{3})$. [5]

6. (a) Define the terms: Euclidean inner product, norm, and distance in $\mathbb{R}^n$. Let $\underline{\boldsymbol{u}} = (\boldsymbol{4}, \boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3})$, $\underline{\boldsymbol{v}} = (\boldsymbol{0}, \boldsymbol{3}, \boldsymbol{8}, -\boldsymbol{2})$, and $\underline{\boldsymbol{w}} = (\boldsymbol{3}, \boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3})$ be vectors in $\mathbb{R}^4$. Evaluate each expression [6]

$$\|\boldsymbol{3}\underline{\boldsymbol{u}} - \boldsymbol{5}\underline{\boldsymbol{v}} + \underline{\boldsymbol{w}}\|, \qquad \left\| \frac{\boldsymbol{1}}{\|\underline{\boldsymbol{w}}\|}\underline{\boldsymbol{w}} \right\|.$$

If $\|\underline{\boldsymbol{u}} + \underline{\boldsymbol{w}}\| = \boldsymbol{1}$ and $\|\underline{\boldsymbol{u}} - \underline{\boldsymbol{w}}\| = \boldsymbol{5}$ then compute $\underline{\boldsymbol{u}} \cdot \underline{\boldsymbol{w}}$.

(b) Define a subspace of a vector space with an example. Does the subset [4]

$$\boldsymbol{W} = \left\{(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}) \in \mathbb{R}^4 \mid \boldsymbol{a} \geq \boldsymbol{b}\right\} \text{ becomes a subspace of } \mathbb{R}^4 \text{ ?}$$

(c) Consider the vectors $\underline{\boldsymbol{u}} = (\boldsymbol{1}, \boldsymbol{2}, -\boldsymbol{1})$ and $\underline{\boldsymbol{v}} = (\boldsymbol{6}, \boldsymbol{4}, \boldsymbol{2})$ in $\boldsymbol{R}^3$. Show that $\underline{\boldsymbol{w}} = (\boldsymbol{9}, \boldsymbol{2}, \boldsymbol{7})$ is a linear combination of $\underline{\boldsymbol{u}}$ and $\underline{\boldsymbol{v}}$ and that $\underline{\boldsymbol{w}}' = (\boldsymbol{4}, -\boldsymbol{1}, \boldsymbol{8})$ is not a linear combination of $\underline{\boldsymbol{u}}$ and $\underline{\boldsymbol{v}}$. [4]

7. (a) Define Image, kernel, rank and nullity of a linear transformation. [4]

(b) Consider the linear transformation $\boldsymbol{T} : \mathbb{R}^3 \to \mathbb{R}^3$ defined by [10]

$$\boldsymbol{T}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = (\boldsymbol{x} + \boldsymbol{2y} - \boldsymbol{z}, \boldsymbol{y} + \boldsymbol{z}, \boldsymbol{x} + \boldsymbol{y} - \boldsymbol{2z})$$

Snow that, $\mathbf{dim}(\mathbf{Im}(\boldsymbol{T})) = \mathbf{Rank}(\boldsymbol{T})$ and $\mathbf{dim}(\mathbf{Ker}(\boldsymbol{T})) = \mathbf{Nullity}(\boldsymbol{T})$.

8. (a) Define eigenvalues and eigenvectors of a matrix. Find a matrix $\boldsymbol{P}$ that diagonalizes [9]

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & -\mathbf{2} \\ \mathbf{1} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{3} \end{bmatrix}.$$

Also, find $\boldsymbol{A}^{\boldsymbol{13}}$.

(b) Prove that the eigenvectors corresponding to distinct eigenvalues are linearly independent. [5]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1^{st} Year 2^{nd} Semester B.Sc (Hons.) Final Examination 2023
Department of Mathematics
MAT 1203: Integral Calculus (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define the antiderivative of a function. Use the antiderivative method to find the area under the curve $y = x^2 + 2x - 5$ over the interval $[0, 2]$. [4]

   (b) $\int_0^4 (|x-2|+1)dx$. Sketch the region whose signed area is represented by the definite integral, and then evaluate the integral using an appropriate formula from geometry, where needed. [4]

   (c) Define definite integration using Riemann Sums. Find the right endpoint approximations of the area under the curve $y = x - x^2$ over the interval $[0, 3]$ with $n = 5$, and $n = 10$. Comment on the accuracy of these approximations. [6]

2. (a) State the fundamental theorem of Calculus. [2]

   (b) Evaluate the following definite integrals [3×4=12]

   (i) $\int_0^{\pi/6} \sec^3 \theta d\theta$

   (ii) $\int_0^{\pi} \frac{5\cos x + 6}{2\cos x + \sin x + 3} dx$

   (iii) $\int_0^1 \frac{x^2+x+1}{\sqrt{1-x^2}} dx$

   (iv) $\int_0^{\pi/2} \frac{dx}{1+\cot x}$

3. (a) State Mean Value Theorem (MVT) for integrals. Verify MVT for the integral [4]

$$\int_1^2 (4 - x^2) dx$$

   (b) Use the Fundamental Theorem of Calculus, to find the value of [4]

$$(i)\ \frac{d}{dx}\left[\int_x^{\pi} \cos\left(\sqrt{t}\right) dt\right] \quad (ii)\ \frac{d}{dx}\left[\int_0^{x} e^{t^2} dt\right]$$

   (c) Derive this Wallis sine formula $\int_0^{\pi/2} \sin^n x dx = \frac{\pi}{2} \cdot \frac{1\cdot 3\cdot 5\cdots(n-1)}{2\cdot 4\cdot 6\cdots n}$ $\left(\begin{array}{c} n \text{ even} \\ \text{and } n \geq 2 \end{array}\right)$. [6]
   And hence evaluate $\int_0^{\pi/2} \sin^8 x dx$.

4. (a) Derive a reduction formula for $\int \sin^n x dx$. Hence evaluate $\int \sin^5 x dx$. [4]

   (b) Sketch the parabola $y^2 = x$. Then find the arc length of the curve from $(0, 0)$ to $(1, 1)$. [4]

   (c) A glass of lemonade with a temperature of $40°F$ is left to sit in a room whose temperature is a constant $70°F$. Using a principle of physics called Newton's Law of Cooling, one can show that if the temperature of the lemonade reaches $52°F$ in 1 hour, then [6] the temperature T of the lemonade as a function of the elapsed time t is modeled by the equation

$$T = 70 - 30e^{-0.5t}$$

where T is in degrees Fahrenheit and t is in hours. This equation conforms to our everyday experience that the temperature of the lemonade gradually approaches the temperature of the room. Find the average temperature T_{ave} of the lemonade over the first 5 hours.

5. (a) Find the length of the arc of the parabola $x^2 = 4ay$, from the vertex to an extremity of the latus rectum. [4]

   (b) Find the area common to the circle $x^2 + y^2 = 4$ and the ellipse $x^2 + 4y^2 = 9$. [4]

   (c) Find the volume of the spindle shaped solid generated by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis. [6]

6. (a) An astronaut's weight (or more precisely, Earth weight) is the force exerted on the astronaut by the Earth's gravity. As the astronaut moves upward into space, the gravitational pull of the Earth decreases, and hence so does his or her weight. If the Earth is assumed to be a sphere of radius **4000 miles**, then it follows from Newton's Law of Universal Gravitation that an astronaut who weighs **150 lb** on Earth will have a weight of [4]

$$w(x) = \frac{2,400,000,000}{x^2}\text{lb}, \quad x \geq 4000$$

   at a distance of x miles from the Earth's center. Use this formula to estimate the work in foot-pounds required to lift the astronaut **220 miles** upward to the International Space Station.

   (b) Use cylindrical shells to find the volume of the solid generated when the region R under $y = x^2$ over the interval $[0, 2]$ is revolved about the line $y = -1$. [5]

   (c) Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}, y = 2$, and $x = 0$ is revolved about the y-axis. [5]

7. (a) Define Gamma and Beta functions. [3]

   (b) For any $m, n > 0$, prove that $\beta(m, n) = \frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$. [5]

   (c) Prove that [6]

$$\int_0^1 \frac{x^2}{(1 - x^4)^{1/2}} dx \cdot \int_0^1 \frac{dx}{(1 + x^4)^{1/2}} dx = \frac{\pi}{4\sqrt{2}}$$

8. (a) Sketch the following polar curve (any three) [3]
   (i) $r = a \quad (a > 0)$ (ii) $r = \sin 3\theta$ (iii) $r = a(1 + \cos\theta)$ (iv) $r^2 = 4\sin 2\theta$

   (b) Express the polar equation [3]

$$r = 2 + \cos\frac{5\theta}{2}$$

   parametrically, and find the minimum revolution number for a complete trace. Evaluate the slope of the tangent line to the above curve at the point where $\theta = \pi/4$.

   (c) Find the area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$. [4]

   (d) The curve $y = \sqrt{4 - x^2}, -1 \leqslant x \leqslant 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about the x-axis. [4]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 2nd Semester B.Sc (Hons.) Final Examination 2023
Department of Mathematics
STA 1205: Introduction to Probability (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define classical probability, empirical probability, sample space, and mutually exclusive events. [4]

   (b) Two coins are tossed. If A is the event "head on the first coin", B is the event "head on the second coin" and C is the event "coins fall alike", show that the events A, B and C are pairwise independent but not completely independent. [4]

   (c) Describe conditional probability. A coin is tossed until a head appears or it has been tossed three times. Given that the head does not appear on the first toss, what is the probability that the coin is tossed three times? [6]

2. (a) State and prove Bayes theorem. [6]

   (b) A group of **20** boys went on trekking. Among them, **5** had altitude sickness, **8** were dehydrated and **10** returned safely without any mishap. What is the probability that [4]
   (i) a boy who had altitude sickness was not dehydrated?
   (ii) a dehydrated boy also had altitude sickness?

   (c) The percentages of students favoring a 4-year honors course in three different universities were as follows: Dhaka University: 21 percent, Rajshahi University: 45 percent and Chittagong University: 75 percent. If a university is chosen at random and a student is selected from this university also at random, what is the probability that the student so selected will be in favor of introducing a 4 -year honors course in the university? Given that the student is in favor of a 4-year course, what is the probability that he comes from Dhaka University? From Chittagong University? From Rajshahi University? [4]

3. (a) If $B_1, B_2, \ldots, B_n$ are mutually exclusive and exhaustive events in a sample space S, then for any event A, prove that [4]

$$P(A) = \sum_{i=1}^{n} P(A \mid B_i) \cdot P(B_i).$$

   (b) Define the probability mass function and probability density function with their properties. [4]

   (c) Suppose X is a random variable with density function [6]

$$f(x) = \begin{cases} \dfrac{k}{(1+x)^2}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

   (i) Find the value of k (ii) Find $F(x)$
   (iii) Find $P(X < 1)$ (iv) Find $P(X > 1)$

4. (a) Define marginal distribution and conditional distribution. [3]

(b) A coin is tossed **3** times. If X denotes the number of heads and Y denotes the number of tails in the last two tosses, find the joint probability distribution of X and Y. [5]

(c) For two continuous random variable X and Y, the joint density is [6]

$$f(x,y) = 2xe^{-y}, 0 \le x \le 1, 0 \le y < \infty.$$

Determine marginal densities of X and Y and hence examine whether X and Y are independent.

5. (a) Define the mathematical expectation of a random variable. Let X denote the number of spots showing on the face of a well-balanced die after it is rolled once. If $Y = X^2 + 2X$, find the variance of X and variance of Y. [4]

(b) Prove that the expected value of the sum of two random variables X and Y is the sum of the expected values of the variables. [4]

(c) Given the following density function [6]

$$f(x) = \begin{cases} 2(x+y-2xy), & 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Determine $E(X), E(Y), E(X+Y)$ and $E(XY)$.
(ii) Also verify whether $E(X+Y) = E(X)+E(Y)$ and $E(XY) = E(X) \cdot E(Y)$.

6. (a) Define the moment-generating function for a random variable. Let X be a random variable with moment generating function $M_X(t)$, then prove that [4]

$$\mu'_r = \frac{d^r}{dt^r} M_x(t) \Big|_{t=0}$$

(b) Suppose X is a random variable for which the probability density function is as follows: [4]

$$f(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0, & \text{elsewhere} . \end{cases}$$

Determine the moment generating function and hence the variance of X.

(c) Describe the cumulant generating function. Establish the relationship between the moment generating function and the cumulant generating function. [6]

7. (a) Define Bernoulli distribution. In a binomial distribution, the mean and the standard deviation are **36** and **4.8**. Find n and p. [4]

(b) Show that the mean and variance of Poisson distribution are equal. [5]

(c) The average number of calls received by a telephone operator during a time interval of **10** minutes daily is **3**. What is the probability that the operator will receive (i) no call and (ii) at least one call tomorrow during the same time interval? [5]

8. (a) Show that the mean, median and mode of the normal distribution are all equal to μ and its variance is σ^2. [6]

(b) Suppose 300 misprints are distributed randomly throughout a book of 500 pages. Find the probability that a given page contains (i) exactly 2 misprints, (ii) 2 or more misprints (iii) no misprints (iv) between 3 and 5 misprints inclusive. [4]

(c) The grade-point average score of 80 students of the Department of Computer Science of Dhaka University in their term-final examination was found to follow approximately a normal distribution with a mean of 2.1 and a standard deviation 0.6 . How many of these students are expected to have a score between 2.5 and 3.5 ? [4]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 2nd Semester B. Sc. (Hons.) Final Examination-2023

Department of Mathematics

PHY 1207: Electricity and Magnetism (3 Credits)

Time: 3 Hours **Full Marks: 70**

Figures shown in the right margin indicate full marks.
Answer 05 out of 08 questions.

1 a. What is electric field? Apply Coulomb's law to find mathematical expression for electric at a distance of d meters from a charge of Q coulombs. [4]

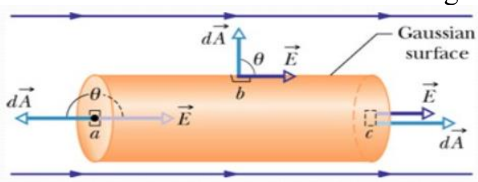
b. What is electric dipole? Use suitable diagram to determine the electric field due to an electric dipole. [6]

c. Find electric field intensity at (3, 4) meters if three-point charges in air are located in a Cartesian coordinate system as follows: [4]
$+5\times10^{-8}$ C at (0, 0) meters, $+4\times10^{-8}$ C at (3, 0) meters and -6×10^{-8} C at (0, 4) meters.

2 a. Use Gauss' law to find mathematical expression of the electric field for charge distributions which have spherical symmetry. [4]

b. Derive mathematical expression to find electrical potential V due to a particle of charge q at any radial distance r from the particle. [6]

c. The following figure shows a Gaussian surface in the form of a closed Gaussian cylinder of radius R. It lies in a uniform electric field with the cylinder's central axis (along the length of the cylinder) parallel to the field. What is the net flux Φ of the electric field through the cylinder? [4]



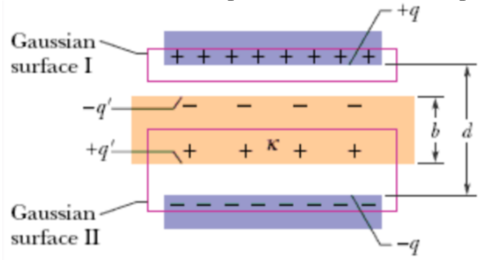
3 a. Calculate the capacitance of a cylindrical capacitor. [4]

b. A parallel plate capacitor is charged by using a battery, which is then disconnected. A dielectric slab is then slipped between the plates. Explain quantitatively what happens to the charge, the capacitance, the potential difference, the electric field, and the stored energy. [5]

c. In a discharging RC circuit, find the resistor's current and potential difference as functions of time [5]

4. a. Derive mathematical expression to determine the equivalent capacitances of several capacitors connected in series with the help of necessary diagram. [4]

b. The following figure shows a parallel-plate capacitor of plate area A = 115 cm^2 and plate separation d =1.24 cm. A potential difference V_0 = 85.5 V is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b = 0.78 cm and dielectric media of dielectric constant k = 2.61 is placed between the plates as shown. [10]

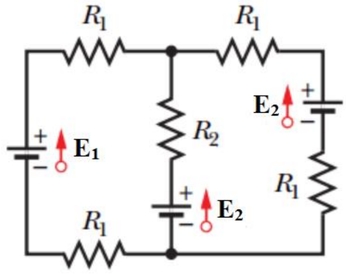


(i) What is the capacitance C_0 before the dielectric slab is inserted?
(ii) What free charge appears on the plates?
(iii) What is the electric field E_0 in the gaps between the plates and the dielectric slab?
(iv) What is the electric field E_1 in the dielectric slab?
(v) What is the potential difference V between the plates after the slab has been introduced?

5 a. What is emf? Draw the circuit diagram of a single-loop circuit and describe energy method to calculate the current in this circuit [4]

b. Three resistors having resistances of R_1, R_2 and R_3 are connected in parallel to an ideal battery of emf E. Derive mathematical expression for the equivalent resistance of the three parallel connected resistors. [4]

c. In the following circuit, $E_1 = 4$ V, $E_2 = 8$ V, $R_1 = 2\ \Omega$ and $R_2 = 4\ \Omega$. Considering ideal batteries, find the magnitude and direction of the current in each of the three branches in the circuit. [6]



6 a Define magnetic field vector B. Find an expression for the radius of the circular path followed by a charged particle circulating in a uniform magnetic field. [4]

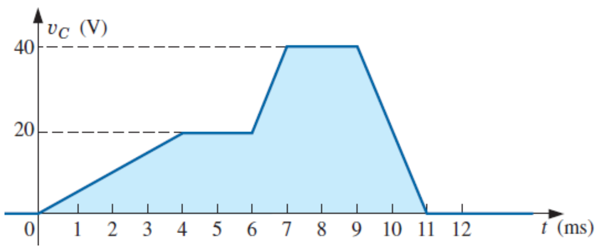
b Derive an expression for force experienced by a current carrying conductor placed in a magnetic field. [4]

c Find an expression for torque produced when a current loop is placed in a uniform magnetic field. [6]

7 a. Define capacitive time constant for a series RC circuit. What happens to the voltage across the plates of a fully-charged capacitor after the first time constant in its discharging phase? [4]

b. Describe the transient behaviors of the voltage v_c across the capacitor and current i_c in a series RC circuit in the charging and discharging phases with the help of necessary diagrams. [6]

c. Determine and draw the waveform for the average current if the voltage across a 2 μF capacitor is as shown below: [4]



8 a. What are the key differences among paramagnetic, diamagnetic, and ferromagnetic materials? [4]

b. What is hysteresis? Why is the hysteresis loop area larger in some ferromagnetic materials than others, and how does this affect their suitability for use in different magnetic applications? [6]

c. How can the Hall effect be used to determine carrier concentration and mobility in a material? [4]

Bangabandhu Sheikh Mujibur Rahman University, Kishoreganj

1st Year 2nd Semester B.Sc (Hons.) Final Examination 2023
Department of Mathematics
MAT 1202: MATHEMATICA Lab (3 Credits)

Time: 03:00 Hours **Full Marks: 60**

N.B.: Answer all the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) If your teacher takes 33 classes, what will be the percentage for doing classes 1 to 33? Find the ceiling value of the percentage and corresponding marks according to the following data [5]

Par	0-59	60-66	67-69	70-72	73-75	76-78	79-81	82-84	85-87	88-90	91-100
Marks	0	1	2	3	4	5	6	7	8	9	10

Print the result in a tabular form with proper headings.

(b) If $\boldsymbol{p}$ dollars are invested for $\boldsymbol{t}$ years in a bank account paying an annual interest rate of $\boldsymbol{r}$ compounded $\boldsymbol{n}$ times a year, the amount of money after $\boldsymbol{k}$ periods is $\boldsymbol{p\left(1+\frac{r}{n}\right)^k}$ dollars. If **\$2000** is invested in an account paying **5%** compounded (quarterly, monthly, and daily) make a table showing how much money has accumulated during a three-year period. [5]

2. (a) Consider the following set of functions: $\boldsymbol{x,\ 2-x^2,\ \sqrt{2-x}.}$ [5]
Plot the above set of curves in the same diagram using different styles. Explain.

(b) Observe the behavior of the graphs of $\boldsymbol{\sin nx,\ \cos nx}$ between $\boldsymbol{-2\pi}$ to $\boldsymbol{2\pi}$ as $\boldsymbol{n}$ changes from **1** to **5**. [5]

3. (a) i) Generate a list containing the angles $\boldsymbol{\theta \in [0, 2\pi]}$ with an increment of $\boldsymbol{\pi/4}$. [5]
ii) Create another list to do the same as in (i) with increment $\boldsymbol{\pi/6}$.
iii) Take the union of the results of (i) and (ii).
iv) Generate another list that contains the values of $\boldsymbol{\theta, \sin\theta}$ and $\boldsymbol{\cos\theta}$ where $\boldsymbol{\theta}$ is the elements of the list obtains in (iii).
v) Construct a table to show the result of (iv) vertically with proper headings.

(b) A palindromic number is a number that remains the same when its digits are reversed. Write a Mathematica code that will print all palindromic numbers between 100 and 2000. [5]

4. (a) Find the limit (if exists) of the following functions at the specified points: [5]

(i) $\boldsymbol{f(x) = \dfrac{x-2}{|x-2|}}$ as $\boldsymbol{x \to 2}$.

(ii) $\boldsymbol{f(x) = \begin{cases} x^2 & \text{for } x \geq 3 \\ 2x & \text{for } x < 3 \end{cases}}$ as $\boldsymbol{x \to 3}$.

(b) Define square Matrices $\boldsymbol{A, B}$ and $\boldsymbol{C}$. Then show that [5]

(i) $\boldsymbol{A(B+C) = AB + AC}$,

(ii) $\boldsymbol{(AB)^{-1} = B^{-1}A^{-1}}$.

5. (a) Consider a second degree equation: [5]

$$f(x,y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$$

(i) Input the values of a, h, b, c, g and f and then test whether the second-degree the equation represents a pair of straight lines or not.

(ii) If the equation in (a) represents a pair of straight lines, then determine the equation of these two lines.

(iii) Then find their point of intersection and angle between the lines.

(iv) Draw the function $f(x, y) = 0$ directly.

(v) Draw the two straight lines in same set of axes using different styles.

(vi) Input $h = 2, b = 1, c = -3, g = -2$ and $f = -1$ and then find the values of "a" so that $f(x, y) = 0$ represents a pair of straight line.

(b) Consider the points $A(-1, 1), B(1, 1)$ and $C(0, 2)$. [5]

(i) Draw the points A, B, and C in the same set of axes and join the points in such a way that they will form a triangle. Store the result in a variable.

(ii) Use the transformation $T(x, y) = (x + 1.5, y + 1.5)$ for each of the vertex of the triangle. Then repeat the process given in (a).

(iii) Show the results of (a) and (b) in the same set of axes.

(iv) Show the result of (a) and its vertical reflection in the same set of axes.

(v) Rotate the triangle shown in (a) by 90 degrees and then draw them in the same set of axes.

6. (a) Consider a third degree polynomial function $f(x) = x^3 - 4x + 1$ defined in $[a, b]$ where $a = -3.25$ and $b = 3.25$. [5]
(i) Generate a real random number, p in the range from $[a, b]$.
(ii) Find the left and right-hand limit of $f(x)$ as $x \to p$. Test whether the limit of $f(x)$ as $x \to p$ exists or not.
(iii) Calculate $f(p)$ and test whether $f(x)$ is continuous at $x = p$ or not.
(iv) Calculate the first and second derivative of $f(x)$ using the following definitions and directly and then store the results in $f1(x)$ and $\mathbf{f2}(x)$ respectively.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2 * f(x) + f(x-h)}{h^2}$$

(b) (i) Observe the behavior of the graph $g(x) = \sin x - \cos(nx)$ between 0 to π as n changes from 1 to 100. [Use Manipulate command] [5]
(ii) Plot the graph of the following function: [Use Piecewise command]

$$h(x) = \begin{cases} -x & \text{if } |x| < 1 \\ \sin x & \text{if } 1 \leq |x| < 2 \\ \cos x & \text{otherwise} \end{cases}$$