Kishoreganj University

1st Year 2nd Semester B.Sc. (Hons.) Final Examination 2024
Department of Mathematics
MAT 1201: Linear Algebra I (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions.
Numbers given in the right margin indicate the marks of the respective questions.

1. (a) What is orthogonal matrix? Prove that if A is an orthogonal matrix then A^{-1} is also orthogonal. [4]

   (b) Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix. [5]

   (c) Prove that every square matrix can be uniquely expressed as P+iQ where P and Q are Hermitian. [5]

2. (a) For which values of k will the following system of equations has exactly one solution, more than one solution, and no solution? [6]

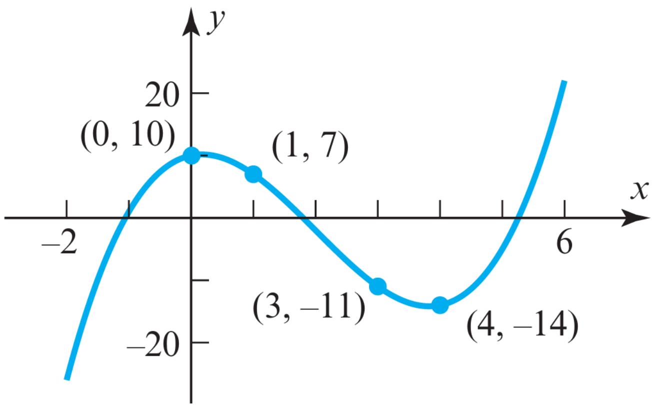
$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + \left(k^2 - 14\right) z &= k + 2. \end{aligned}$$

   (b) Balancing chemical equations using linear systems [8]

$$HCl + Na_3PO_4 \rightarrow H_3PO_4 + NaCl$$

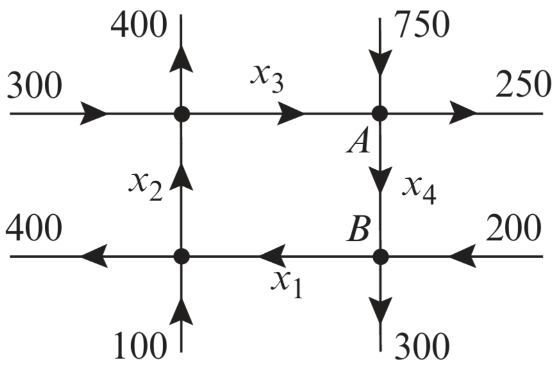
[hydrochloric acid] + [sodium phosphate] → [phosphoric acid] + [sodium chloride]

3. (a) Find the coefficients a, b, c, and d so that the curve shown in the accompanying figure is the graph of the equation $y = ax^3 + bx^2 + cx + d$. [7]



   (b) The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour. [7]

   i. Set up a linear system whose solution provides the unknown flow rates.
   ii. Solve the system for the unknown flow rates.



4. (a) Using inversion algorithm find the inverse of the following matrix if inverse exists. [6]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

(b) When a matrix is said to be invertible? Prove that if a matrix A is invertible then its transpose is also invertible. [4]

(c) Find all values of x, if any, for which the given matrix is invertible. [4]

$$A = \begin{bmatrix} x & x & x \\ 1 & x & x \\ 1 & 1 & x \end{bmatrix}$$

5. (a) When a non empty set become a vector space? [3]

(b) Prove that the space of 2×2 matrices is a vector space [6]

(c) Determine whether the vectors $v_1 = (2,-1,3)$, $v_2 = (4,1,2)$, $v_3 = (8,-1,8)$ span R^3. [5]

6. (a) Determine whether the vectors $2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$ are linearly independent or are linearly dependent in P_2. [5]

(b) What is basis? Show that the set of vectors (3, 1, 4), (2, 5, 6), (1, 4, 8) forms a basis for R^3. . [6]

(c) Find a basis for the solution space of the homogeneous linear system, and find the dimension of that space. [3]

$$3x_1 + x_2 + x_3 + x_4 = 0$$
$$5x_1 - x_2 + x_3 - x_4 = 0$$

7. (a) Define eigenvalues and eigenvectors of a matrix. Find a matrix P that diagonalizes [10]

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}.$$

Also, find A^{13}.

(b) The invertible matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ maps the line $y = 2x + 1$ into another line. Find its equation. [4]

8. (a) State and prove the Cayley-Hamilton theorem. [7]

(b) Verify the Cayley-Hamilton theorem for the matrix [7]

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

and hence find A^{-1}.

Kishoreganj University

1^{st} Year 2^{nd} Semester B.Sc. (Hons.) Final Examination 2024
Department of Mathematics
MAT 1203: Integral Calculus (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

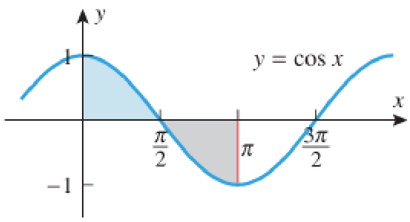
1. (a) Define antidifferentiation and antiderivative method. Use the antiderivative method to find the area under the graph of $y = x^2$ over the interval $[0, 1]$. [4]

   (b) Suppose that a curve $y = f(x)$ in the xy-plane has the property that at each point (x, y) on the curve, the tangent line has slope x^2. Find an equation for the curve given that it passes through the point $(2, 1)$. [4]

   (c) Evaluate $(i) \int t^4(3-5t^5)^{\frac{1}{3}}\, dt, \quad (ii) \int_{-1}^{2} |2x-3|\, dx, \quad (iii) \int_2^5 (2x-5)(x-3)^9\, dx.$ [6]

2. (a) State and prove the Fundamental Theorem of Calculus, Part 1. [6]

   (b) The $\cos x$ figure is shown below, and answer the following question. [4]



   (i) Find the area under the curve $y = \cos x$ over the interval $[0, \pi/2]$ (figure).

   (ii) Make a conjecture about the value of the integral $\int_0^{\pi} \cos x\, dx$ and confirm your conjecture using the Fundamental Theorem of Calculus.

   (c) State the Mean-Value Theorem for Integrals. Verify the function $f(x) = x^2$ is continuous on the interval $[1, 4]$ by the Mean-Value Theorem for Integrals. [4]

3. (a) State the Fundamental Theorem of Calculus, Part 2. Find $\frac{d}{dx}\left[\int_1^x t^3\, dt\right]$ by applying Part 2 of the Fundamental Theorem of Calculus, and then confirm the result by performing the integration and then differentiating. [5]

   (b) A bus has stopped to pick up riders, and a woman is running at a constant velocity of 5 m/s to catch it. When she is 11 m behind the front door, the bus pulls away with a constant acceleration of 1 m/s^2. From that point in time, how long will it take for the woman to reach the front door of the bus if she keeps running with a velocity of 5 m/s? [4]

   (c) Define mean value. Find the mean value of the function $f(x) = \sqrt{x}$ over the interval $[1, 4]$, and find all points in the interval at which the value of f is the same as the mean. [5]

4. (a) Find the reduction formula for $\int \cos^n x\, dx$. [3]

   (b) If $I_n = \int_0^{\pi/4} \tan^n \theta\, d\theta$ then show that, $n(I_{n+1} + I_{n-1}) = 1$ and from this find the value of $\int_0^{\pi/4} \tan^6 \theta\, d\theta$. [5]

(c) If n be a positive integer then show that [6]

$$I = \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx$$
$$= \begin{cases} \dfrac{(n-1)(n-3)(n-5)...5.3.1}{n(n-2)(n-4)...6.4.2} \cdot \dfrac{\pi}{2} & \text{when } n \text{ is even,} \\ \dfrac{(n-1)(n-3)(n-5)...6.4.2}{n(n-2)(n-4)...5.3.1} & \text{when } n \text{ is odd.} \end{cases}$$

5. (a) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$. [4]
   (b) Find the volume of the solid generated when the region under the curve $y = x^2$ over the interval $[0, 2]$ is rotated about the line $y = -1$. [6]
   (c) Define arc length and find the arc length of the curve $y = x^{3/2}$ from $(1, 1)$ to $(2, 2\sqrt{2})$. [4]

6. (a) Evaluate the following improper integral [8]

$$(i) \int_0^{\pi/2} \frac{dx}{1 - \sin x}, \quad (ii) \int_{-\infty}^{\infty} \frac{dx}{1 + x^2}, \quad (iii) \int_0^4 \frac{dx}{x\sqrt{x}}, \quad (iv) \int_3^4 \frac{dx}{\sqrt{x - 3}}.$$

   (b) Test the convergence [6]

$$(i) \int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}, \quad (ii) \int_0^1 \frac{dx}{\sqrt{x(1 - x)}}.$$

7. (a) Define Gamma function and Show that $\Gamma(n + 1) = n\Gamma(n)$. [4]
   (b) Evaluate $\int_0^{\infty} \frac{x^a}{a^x} dx$, hence show that $\int_0^{\infty} \frac{x^7}{7^x} dx = \frac{7!}{(\log 7)^8}$, $(a > 1)$. [4]
   (c) Find the value of $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1 + x)^{m+n}} dx.$ [6]

8. (a) Express the polar equation [6]

$$r = 2 + \cos\left(\frac{5\theta}{2}\right)$$

   parametrically, and generate the polar graph from the parametric equations using a graphing utility.
   (b) Find the slope of the tangent line to the circle $r = 4\cos\theta$ at the point where $\theta = \pi/4$. [4]
   (c) Find the area of the region in the first quadrant that is within the cardioid $r = 1 - \cos\theta$. [4]

Kishoreganj University

1^{st} Year 2^{nd} Semester B.Sc. (Hons.) Final Examination 2024
Department of Mathematics
STA 1205: Introduction to Probability (3 Credits)

Time: 03:00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Write down the difference between event and outcome of an experiment. [4]

(b) If $P(A) = 0.8$, $P(B) = 0.5$, and $P(A \cap B) = 0.4$, determine the following probabilities: $P(A \cup B)$, $P(A^c \cap B)$ and $P(A^c \cup B)$. [4]

(c) Ideal number of children in a family is two. If having a boy and having a girl are equally likely independent events, find the probability that a family has (i) two girls; (ii) one boy and one girl; (iii) at least one girl. [6]

2. (a) State and prove Bayes theorem. [7]

(b) An opinion poll in Kishoreganj city shows that **45** percent of the city dwellers support NCP (A), **40** percent support the Bangladesh Nationalist Party (B) and the remaining **15** percent support Communist and other parties (C). Previous records reveal that in city elections, **65** percent of the NCP, **80** percent of the Bangladesh Nationalist Party and **50** percent of the Communist or other party supporters turned up to cast their votes. A person in the City is chosen at random and it is learned that he did not cast his vote in the last election. What is the probability that he is a supporter of A ? B ? C ? [7]

3. (a) Define the probability mass function and probability density function with their properties. [4]

(b) The probability function of a discrete random variable X is [5]

$$f(x) = \begin{cases} a\left(\frac{3}{4}\right)^x; x = 0, 1, 2, \ldots . . \\ 0; \text{ elsewhere} \end{cases}$$

Evaluate α. Also find $p(X \leq 3)$.

(c) A continuous random variable X has the following density function: [5]

$$f(x) = \begin{cases} \frac{2}{27}(1+x), \quad 2 < x < 5 \\ 0; \text{ elsewhere} \end{cases}$$

(i) Find $P(X < 4)$ and (ii) Find $P(3 < X < 4)$.

4. (a) Describe marginal probability distribution. Find the marginal densities of X and Y from the following joint density function and verify that marginal distributions are also probability distributions. [6]

$$f(x, y) = \begin{cases} \frac{1}{8}(6 - x - y), & \text{for } 0 < x < 2,\ 2 < y < 4 \\ 0, & \text{otherwise} \end{cases}$$

Also compute $P(X + Y < 3)$ and $P(X < 1.5, Y < 2.5)$.

(b) Define conditional probability distribution. Given the following joint distributions of the discrete random variables X and Y: [5]

	X		
Y	0	1	2
0	3/28	9/28	3/28
1	6/28	6/28	0
2	1/28	0	0

Find $f(x \mid 1)$, $f(y \mid 1)$ and $P(X = 0 \mid Y = 1)$.

(c) Consider the following density function: [3]

$$f(x, y) = \begin{cases} 6xy^2, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}.$$

Show that X and Y are independent.

5. (a) Define mathematical expectation. Suppose that X represents the number of defective teeth of a patient visiting a certain dental clinic. The pmf of X is given as: [4]

X	1	2	3	4	5
$f(X)$	0.25	0.35	0.20	0.15	k

(a) Find the value for k.

(b) Find the expected number of defective teeth of a patient.

(b) The hospital stay in days for patients following treatment for a certain type of kidney disorder is a random variable defined as $Y = X + 4$, where the probability density function of X is [4]

$$f(x) = \frac{32}{(x+4)^3}, \quad x > 0.$$

Find the expected number of days that a patient has to stay at the hospital after taking such treatment.

(c) Suppose that X is a continuous random variable with pdf [6]

$$f(x) = \frac{1}{\lambda} e^{-\frac{1}{\lambda}x}, \quad x > 0,$$

where $\lambda > 0$ is a constant. This is known as exponential distribution with parameter λ. Find the moment generating function of X. Hence find the population mean and variance.

6. (a) Define Binomial probability distribution. Find the mean and variance of the Binomial distribution. [5]

(b) Derive the MGF and then drive the CGF for Binomial probability distribution. [5]

(c) If X is a binomial variable with $n = 5$ and $p = 0.3$, evaluate the following: [4]
(i) $P(X \le 4)$ (ii) $P(X = 2)$ (iii) $P(X < 3)$ (iv) $P(X > 1)$.

7. (a) What is a Poisson distribution? Give some examples of a Poisson variable. [4]

(b) Derive the MGF of Poisson distribution, and then find the mean and variance from MGF. [5]

(c) Suppose that there are, on the average, four vehicle accidents per day on the highways running from Dhaka to Kishoreganj. What is the probability that on a given day in the highways: (i) There is no vehicle accident? (ii) There are three or fewer accidents? (iii) There are three or more accidents? [5]

8. (a) Define normal distribution. State the properties of normal distribution. [5]
Verify that the area under the normal curve is 1.

(b) Phillips Bangladesh manufactures electric bulbs that have a length of life that is normally distributed with a mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. [4]

(c) Given the normally distributed variable X with a mean 18 and standard deviation 2.5. Find the value of k such that $P(X < k) = 0.2578$. [5]

Kishoreganj University

1st Year 2nd Semester B.Sc. (Hons.) Final Examination 2024

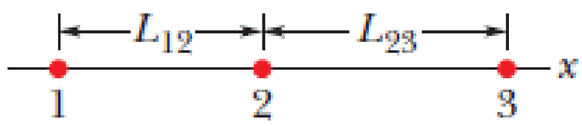
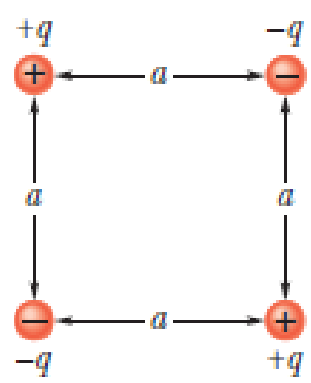
Department of Mathematics

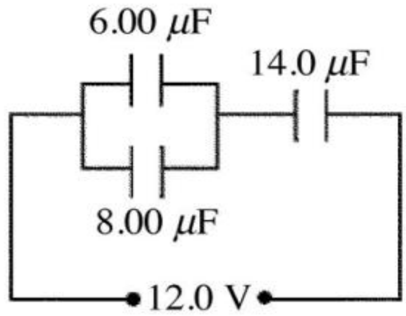
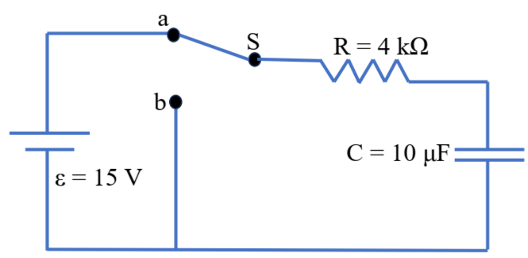
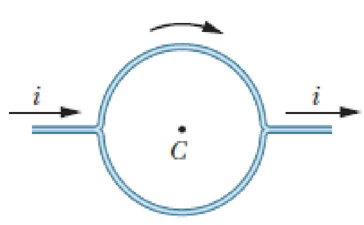
PHY 1207: Electricity & Magnetism (3 Credits)

Full Marks: 70, Time: 3 hours

Answer any five (5) out of eight (8) questions

(Figures shown in the right margin indicate full marks)

1. a)	What do you understand by quantization of electric charges? Write Coulomb's law in vector form and explain the meaning of each term.	**(4)**
b)	Find the expression of electric field E (at point P) due to a point charge q located at a distance r from the point P. How do you calculate E (at same point) in case of group of point charges instead of a single point charge?	**(6)**
c)	In the Fig, three charged particles lie on an x axis. Particles 1 and 2 are fixed in place. Particle 3 is free to move, but the net electrostatic force on it from particles 1 and 2 happens to be zero. If $L_{23} = L_{12}$, what is the ratio q_1/q_2? 	**(4)**
2. a)	What is electric flux? Show that the dimensions of electric flux is $[ML^3T^{-3}A^{-1}]$, where the symbols have their usual meanings.	**(4)**
b)	State Gauss law in electrostatics. Derive the relation $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$, where is the volume density of the charge.	**(6)**
c)	An electric flux of 158 $N.m^2/C$ passes through a flat horizontal surface that has an area of 0.62 m^2. The flux is due to a uniform electric field. What is the magnitude of the electric field if the field points 15 degrees above the horizontal?	**(4)**
3. a)	How do you differentiate between electric potential and potential difference?	**(3)**
b)	Show that the potential V at an arbitrary point due to an electric dipole is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$, where the symbols have their usual meanings.	**(7)**
c)	Four charged particles are arranged as shown. If $q = 2.50\ \mu C$ and $a = 10\ cm$, what is the value of net electric potential at the center of the square? 	**(4)**
4. a)	Define capacitor and capacitance of a capacitor.	**(4)**
b)	Find an expression for the capacitance of a spherical capacitor, consisting of concentric spherical shells of radii a (inner shell) and b (outer shell).	**(6)**
c)	Two capacitors of capacitance 6.00 μF and 8.00 μF are connected in parallel. The combination is then connected in series with a 12 V voltage source and a 14.0 μF capacitor as shown in the figure.	**(4)**

	 (i) Calculate the charge on the 14.0 μF capacitor. (ii) What is the potential difference across the 6.00 μF capacitor? (iii) What is the charge on the 8.00 μF capacitor?	
5. a)	State and Explain Kirchhoff's current law and voltage law.	**(4)**
b)	Derive the charging and discharging equations for the voltage readings across the resistor and capacitor of a RC- circuit.	**(6)**
c)	A charging and discharging RC circuit consist of a resistor, a capacitor and a battery as shown in the diagram. Here, 4 $k\Omega$, $C = 10\ \mu F$ and $\varepsilon = 15\ V$.  (i) Determine the time constant (ii) Calculate the maximum charge on the capacitor could acquire after capacitor is fully charged (switch is at a) (iii) At $t = 0$ the switch is thrown from position a to b to make a discharging circuit. Calculate the charge on the capacitor at $t = 10\ ms$.	**(4)**
6. a)	What is RC circuit? Explain the physical significance of the time constant of an *RC* circuit.	**(2+2)**
b)	Derive the differential equation for charging of a capacitor in *RC* circuit and find the expression of charging current.	**(6)**
c)	Plot the graph of discharging current as a function of time up to four-time *constants* of *RC* circuit.	**(4)**
7. a)	How do you define solenoid and toroid?	**(4)**
b)	Calculate the magnetic field ***B*** for a solenoid and show that *B* inside the solenoid (along the axis) does not depend on the length or the diameter of the solenoid.	**(6)**
c)	A solenoid *1.30 m* long and *2.60 cm* in diameter carries a current of *18.0 A*. The magnetic field inside the solenoid is *23.0 mT*. Find the length of the wire forming the solenoid.	**(4)**
8. a)	State and explain Ampere's law for magnetic field.	**(2)**
b)	Show that the magnetic field inside a long straight wire (of radius *R*) with a current *I* is $B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$, where the symbols have their usual meanings. At which point (along the radial distance) of the wire will have minimum and maximum magnetic fields? Sketch a graph for *B* versus *r* for the range of $r = 0$ to $r >> R$.	**(5+3)**
c)	A straight conductor carrying current $i = 5.0\ A$ splits into identical semicircular arcs as shown in Fig. below. What is the magnetic field at the center *C* of the resulting circular loop? 	**(4)**

Kishoreganj University

1^{st} Year 2^{nd} Semester B.Sc. (Hons.) Final Examination 2024
Department of Mathematics
MAT 1202: MATHEMATICA Lab (3 Credits)

Time: 03:00 Hours **Full Marks: 60**

N.B.: Answer all SIX (6) questions.
Numbers given in the right margin indicate the marks of the respective questions.

1. (a) If p dollars is invested for t years in a bank account paying an annual interest rate of r compounded n times a year, the amount of money after k periods is $p(1+\frac{r}{n})^k$ dollars. If \$2000 is invested in an account paying 5% compounded (quarterly, monthly and daily) make a table showing how much money has accumulated during a three-year period. [5]

   (b) (i) For the numbers $12, 24$ and 28, find the divisors and sum of the divisors. Are these numbers perfect? [5]
   (ii) How many prime numbers between 1 and 150?
   (iii) For the first 30 positive integers separate primes from non primes.
   (iv) Print all numbers from 1 to 30 which are not multiples of $2, 3$, or 5.
   (v) How many numbers from 1 to 60 which are divisible by 13? Print these numbers.

2. (a) Plot the graphs of $y = cosx$, $y = cos^{-1}x$ and $y = x$ in the same diagram and use different styles to identify the graphs. [5]

   (b) Plot the followings: [5]
   (i) $y = \frac{1}{|4-x^2|}$;
   (ii) $y = \sqrt{1-x} + \sqrt{x-1} + 0.5$;
   (iii) $r = a + bsin\theta$, for $\frac{a}{b} < 1$, $\frac{a}{b} = 1$, $1 < \frac{a}{b} < 2$, $\frac{a}{b} \geq 2$;
   (iv) $r = sin2\theta - cos3\theta$.

3. (a) i) The Fibonacci sequence is defined by $f_0 = 1, f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$. Define the Fibonacci function. [6]
   ii) Use the definition in (i) to generate a list of the first 300 values of the sequence.
   iii) Use the result of (ii) to create a list of numbers which are prime numbers as well as Fibonacci numbers.

   (b) Determine all positive integers n between 0 and 60 for which n^2+n+41 is a prime number. [4]

4. (a) If $y = \text{Sin}\left(m\,\text{Sin}^{-1}\,x\right)$ then show that [5]

$$\left(1-x^2\right)y_4 - 5xy_3 + \left(m^2-4\right)y_2 = 0.$$

   (b) Find the following integrals: (i) $\int \frac{(3x+2)}{\sqrt{x^2+4x+2}}dx$ (ii) $\int_0^{\pi/4} \frac{\text{Sin}\,2x}{\text{Sin}^4\,x+\text{Cos}^4\,x}dx$. [5]

5. (a) Consider a second degree equation: [5]

$$f(x,y) \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

   (i) Input the values of a, h, b, c, g and f and then test whether the second degree equation represents a pair of straight line or not.
   (ii) If the equation in (i) represents a pair of straight line, then determine the equation of these two lines.
   (iii) Then find their point of intersection and angle between the lines.
   (iv) Draw the function $f(x,y) = 0$ directly.
   (v) Draw the two straight lines in same set of axes using different styles.

(b) (i) Input the value of p, a, b and c where $a \neq b$. [5]
(ii) Find the focus, vertices and directrix of the parabola $y^2 = 4px$.
(iii) Sketch the parabola, focus and vertices in same set of axes.
(iv) Find the focus, vertices and directrix of the ellipse $ax^2 + by^2 = c^2; a \neq b$.
(v) Sketch the ellipse, foci and vertices in same set of axes.

6. (a) Consider a third degree polynomial function $f(x) = x^3 - 4x + 1$ defined in $[a, b]$ where $a = -3.25$ and $b = 3.25$. [6]
(i) Calculate the first and second derivative of $f(x)$ using the following definitions and directly and then store the results in $f1(x)$ and f2(x) respectively.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2 * f(x) + f(x-h)}{h^2}$$

(ii)Draw the graph of $f(x), f1(x)$ and $f2(x)$ using different colors and styles in the same set of axis and store the result in "gs1".
(iii) Find the critical point of $f(x)$ by solving $g(x) = 0$.
(ivi) Find the inflection point of $f(x)$ by solving $h(x) = 0$.
(v) Show the result of (v), the point (p,f(p)) and both the critical and inflection points in the same set of axes. Also, use heavy dots and different colors for these points.

(b) Consider the functions [4]

$$q1(x) = x(x-1)(x-2) \text{ and } q2(x) = (x-1)(x-2)(x-3)$$

and defined in $[0, 4]$.
(i) Draw the graph of the functions in the same set of axes using different styles.
(ii) Find the solution of the following equation

$$\frac{q1[b] - q1[a]}{q2[b] - q2[a]} = \frac{q1'[c]}{q2'[c]}$$

with respect to c .