Kishoreganj University

2^{nd} Year 2^{nd} Semester B.Sc. (Hons.) Final Examination-2024
Department of Mathematics
MAT 2201: Linear Algebra II (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define linear functional and dual space with examples. Let $\{v_1, \ldots, v_n\}$ and $\{w_1, \ldots, w_n\}$ be bases of V and let $\{\phi_1, \ldots, \phi_n\}$ and $\{\sigma_1, \ldots, \sigma_n\}$ be the bases of V^* dual to $\{v_i\}$ and $\{w_i\}$, respectively. Suppose P is the change-of-basis matrix from $\{v_i\}$ to $\{w_i\}$. Then $(P^{-1})^T$ is the change-of-basis matrix from $\{\phi_i\}$ to $\{\sigma_i\}$. Prove that $Q = \left(P^T\right)^{-1} = (P^{-1})^T$. [7]

   (b) Let $H = \begin{bmatrix} 1 & 1+i & 2i \\ 1-i & 4 & 2-3i \\ -2i & 2+3i & 7 \end{bmatrix}$, a Hermitian matrix. Find a nonsingular matrix P such that $D = P^T H \bar{P}$ is diagonal. Also, find the signature of H. [7]

2. (a) What is an annihilator? Suppose V has finite dimension and W is a subspace of V. Then show that (i) $\dim W + \dim W^0 = \dim V$ and (ii) $W^{00} = W$. [6]

   (b) Find the basis $\{\phi_1, \phi_2, \phi_3\}$ that is dual to the following basis of $\mathbf{R}^3$: [4]

$$\{v_1 = (1, -1, 3), \quad v_2 = (0, 1, -1), \quad v_3 = (0, 3, -2)\}.$$

   (c) Let ϕ be the linear functional on $\mathbf{R}^2$ defined by $\phi(x, y) = x - 2y$. For each of the following linear operators T on $\mathbf{R}^2$, find $(T^t(\phi))(x, y)$: (i) $T(x, y) = (x, 0)$, (ii) $T(x, y) = (y, x+y)$, (iii) $T(x, y) = (2x - 3y, 5x + 2y)$. [4]

3. (a) Define normal operator, unitary operator and positive operator on an inner product space. If T_1 and T_2 are normal operators then show that $T_1 + T_2$ and $T_1 T_2$ are also normal. [4]

   (b) Let V be a finite dimensional inner product space and T be a normal operator on V. Suppose $\alpha \in V$ be a vector. Then prove that α is a characteristic vector for T with characteristic value c if and only if α is a characteristic vector for the adjoint T^* of T with characteristic value $\bar{c}$, the conjugate of c. [5]

   (c) For the real symmetric matrix [5]

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -1 \end{pmatrix}$$

   find a non-singular matrix P such that $P^t A P$ is diagonal where P^t denotes the transpose of P. Also find its signature.

4. (a) Derive the projection matrix when a line is projected on a plane. [6]

   (b) The table shows the world population (in billions) for six different years. (Source: U.S. Census Bureau) [8]

Year	1980	1985	1990	1995	2000	2005
Population (y)	4.5	4.8	5.3	5.7	6.1	6.5

Let $x = 0$ represent the year 1980. Find the least squares regression quadratic polynomial $y = c_0 + c_1 x + c_2 x^2$ for these data and use the model to estimate the population for the year 2010 .

5. (a) Find the third order Fourier approximation of $f(x) = x$ on $[0, 2\pi]$. [7]

   (b) Consider the following bases of $\mathbf{R^2}$: [7]
   $S = \{u_1 = (1, -2), u_2 = (3, -4)\}$ and $S' = \{v_1 = (1, 3), u_2 = (3, 8)\}$.
   (i) Find the change-of-basis matrix P from S to S'.
   (ii) Find the change-of-basis matrix Q from S' back to S.
   (iii) Verify $Q = P^{-1}$.

6. (a) Let f be any bilinear form on a vector space V and suppose [7]

$$g(x, y) = \frac{1}{2}\big[f(x, y) + f(y, x)\big] \quad \text{and} \quad h(x, y) = \frac{1}{2}\big[f(x, y) - f(y, x)\big].$$

   Prove that g is symmetric and h is skew-symmetric bilinear form and f is their sum.

   (b) Obtain the linear transformation which reduces the quadratic form [7]

$$2x_1^2 + 2x_2^2 + 3x_3^2 - 4x_2x_3 - 4x_1x_3 + 2x_2x_1$$

   to the sum of squares and hence reduce it to the canonical form. Determine its rank, index and signature. Is the given quadratic form positive definite? Give reason in support of your contention.

7. (a) Define positive definite. Write down the different test procedures to determine the positive definite of a matrix. [4]

   (b) Prove that if $1 + 1 \neq 0$ in K, then the bilinear form f can be obtained from the quadratic form q by the polar form of f : [4]

$$f(u, v) = \frac{1}{2}[q(u + v) - q(u) - q(v)].$$

   (c) Determine the definiteness of the quadratic form $Q(x, y) = 3x^2 - 2xy + 4xz + 3y^2 - 4yz + 2z^2$ on $\mathbb{R}^3$. [6]

8. (a) Define Jordan canonical form of a matrix. Let [8]

$$A = \begin{pmatrix} 5 & 1 & -2 & 4 \\ 0 & 5 & 2 & 2 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix},$$

   then find an invertible matrix P such that $P^{-1}AP$ is in Jordan canonical form.

   (b) What do you mean by Singular Value Decomposition (SVD) of an $m \times n$ matrix A? Find an SVD of the matrix [6]

$$A = \begin{pmatrix} 2 & -1 \\ 2 & 2 \end{pmatrix}.$$

Kishoreganj University

2^{nd} Year 2^{nd} Semester B.Sc. (Hons.) Final Examination-2024
Department of Mathematics
MAT 2202: FORTRAN Lab (3 Credits)

Time: 03.00 Hours **Full Marks: 60**

N.B.: Answer all the following questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Print the first n **Happy** numbers. [5]

   (b) Write a program to determine whether a given number is a **Keith** number or not. [5]

   (c) Determine the sum of all perfect numbers between 100 and 400. [5]

2. (a) Input N numbers and display them in ascending and descending order using a function. [6]

   (b) Write a FORTRAN program to find, print, and count all ordered pairs of integers (x, y) that satisfy the equation: $xy = 360$. [5]

   (c) Write a FORTRAN code to determine whether A, B, C can form a triangle. If not, print the message 'Not a Triangle'. If yes, determine whether A, B, C constitute the sides of [4]

   (i) an equilateral triangle.

   (ii) a right–angled triangle.

3. (a) Read a matrix A of order n, print the sum of: [4]
   (i) diagonal elements,
   (ii) the last column elements.

   (b) The commissions of an agent are as follows: [6]

   (i) If $0 < \text{sales} \leq 150$, then commission is 3% of the sales.

   (ii) If $151 \leq \text{sales} \leq 400$, then commission is 6% of the sales.

   (iii) If $401 \leq \text{sales} \leq 800$, then commission is 8% of the sales.

   (iv) sales > 800, then commission is 12% for the first $800 and 15% for the rest.

   Complete the following table by calculating the commissions of the agent for the given sales.

Day	Sales	Commissions
Monday	120	?
Tuesday	450	?
Wednesday	835	?
Thursday	999	?
Friday	310	?
Total Commissions		?

   (c) Compute the sum and the product of the first n terms of the following series: $1, \quad 1-\frac{1}{2}, \quad 1-\frac{1}{2}+\frac{1}{3}, \quad 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}, \cdots$. [5]

4. (a) Consider Wallis' equation [4]

$$f(x) = x^3 - 2x - 5 = 0.$$

   Use Bisection Method to find the real root of the Wallis' equation.

(b) From the following table of yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46, 56 and 69 by using Newton's interpolation formula: [5]

Age X	45	50	55	60	65	70
Premium Y	114.84	96.16	83.32	74.48	68.48	60.33

(c) Evaluate the definite integral [6]

$$\int_0^1 \frac{1}{1+x^2} dx$$

by using

(i) Trapezoidal rule,
(ii) Simpson's 1/3 rule,
(iii) Simpson's 3/8 rule,
correct up to 5 decimal places. Compare your results with the exact value to identify the best rule of numerical integration.

Kishoreganj University

2^{nd} Year 2^{nd} Semester B.Sc. (Hons.) Final Examination-2024
Department of Mathematics
MAT 2203: Integral and Vector Calculus (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define the double integral. Use a double integral to find the area of the portion between the chord joining the points $(-1, 1)$ to $(8, 4)$ and the semi-cubical parabola $y^3 = x^2$. [6]

   (b) Determine $\iint_R y\ dA$ over the region R enclosed by $y^2 = 4x,\ x^2 = 4y,\ x = 3$ and $x + y = 3$. [4]

   (c) Use the change of variables $u = \dfrac{y}{x}, v = xy$ to evaluate $\iint_R xy^3\ dxdy$, where R is the region enclosed by $y = x,\ y = 3x,\ xy = 1,\ xy = 4$. [4]

2. (a) Define simple polar region and polar rectangle with an example. How to find the limits of integration of $f(r, \theta)$ over the region R that inside the cardioid $\mathrm{r} = 2+\cos\theta$ and outside the circle $\mathrm{r} = 2$. [6]

   (b) Use a double integral to find the area enclosed by the three petaled rose $r = \sin 3\theta$. [4]

   (c) Find the polar moment of inertia about the origin of a thin plate of density $\delta(x, y) = 1$ bounded by the quarter circle $x^2 + y^2 = 1$ in the first quadrant. [4]

3. (a) Evaluate $\iiint_G xyz\ \ dv$, where G is the solid in the first octant bounded by the parabolic cylinder $z = 2 - x^2$ and the planes $z = 0,\ y = x,\ y = 0$. [4]

   (b) Utilize cylindrical coordinates to evaluate the integral $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} x^2\ dv$. [5]

   (c) Use spherical coordinates to evaluate the integral [5]

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{4-x^2-y^2} z^2 \sqrt{x^2 + y^2 + z^2}\ dv.$$

4. (a) Determine the surface area of the sphere $x^2 + y^2 + z^2 = a^2$. [4]

   (b) Find the volume of the portion of the cylinder $\dfrac{x^2}{a^2} + \dfrac{z^2}{c^2} = 1$ lying between two planes $y = 0$ and $y = mx$. [6]

   (c) Find the average value of the function $f(x, y) = x \cos xy$ over the rectangular region $R = \{(x, y) : 0 \leq x \leq \pi,\ 0 \leq y \leq 1\}$. [4]

5. (a) Define triple integrals in cylindrical coordinates and spherical coordinates. Write down their relationship with cartesian coordinates. Plot the point with cylindrical coordinates $(2, 2\pi/3, 1)$ and spherical coordinates $(2, \pi/4, \pi/3)$. Also find their rectangular coordinates. [8]

   (b) Use spherical coordinates to find the volume of the solid that above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$. [6]

6. (a) State and prove Green's theorem. [7]

(b) Verify Green's theorem in the plane for $\oint_c (2x - y^3)\ dx - xy\ dy$ where c is the boundary of region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. [7]

7. (a) Define line integral. If $\mathbf{A} = (2y+3)\mathbf{i} + xz\mathbf{j} + (yz - x)\mathbf{k}$, then show that $\int_c \mathbf{A}\cdot d\mathbf{r} = 8$ along the path c, the straight line joining $(0,0,0)$ and $(2,1,1)$. [5]

(b) Define surface integral. Show that $\iint\limits_S \phi \mathbf{n} ds = 100(\mathbf{i} + \mathbf{j})$, where $\phi = \frac{3}{8}xyz$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. [6]

(c) Prove that $\oint_c \phi\ d\mathbf{r} = \iint\limits_s d\mathbf{s} \times \nabla\phi$. [3]

8. (a) Prove that, [6]

$$\iiint_v \frac{1}{r^2} dV = \iint_s \frac{\vec{r} \cdot \vec{n}}{r^2} ds.$$

(b) State Stoke's theorem. Verify Stokes' theorem for $\mathbf{A} = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. [8]

Kishoreganj University

2^{nd} Year 2^{nd} Semester B.Sc. (Hons.) Final Examination-2024
Department of Mathematics
MAT 2205: Numerical Analysis I (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define round-off error and truncation error. What is the difference between them? Also, find the absolute, relative and percentage errors of the number 8.6 if both of its digits are correct. [5]

   (b) What do you mean by bracketing interval? Describe the bisection method to find a real root of $f(x) = 0$. [5]

   (c) Using Newton-Raphson method, derive a formula for for finding the $k-$th root of of a positive number N and hence compute the value of $25^{\frac{1}{4}}$. [4]

2. (a) Write the different stopping criteria for finding numerical roots. Discuss Fixed point iteration method for finding a real root of the equation $f(x) = 0$. [6]

   (b) A floating ball has a specific gravity of 0.6 and has a radius of 5.5 cm . The equation that gives the depth x to which the ball is submerged under water is given by [8]

$$f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4}.$$

   Use the secant method of findings roots of equations to find the depth x to which the ball is submerged under water.
   i) Conduct three iterations to estimate the root of the above equation.
   ii) Find the relative approximate error.

3. (a) Define interpolation and extrapolation. Derive the Newton's formula for the forward difference interpolation. [6]

   (b) The population of a town in decennial census were given in the following table: [4]

Year	1951	1961	1971	1981	1991
Population (in thousand)	46	66	81	93	101

   Estimate the population for the year 1985 using Newton's backward formula.

   (c) Derive the Lagrange's interpolation formula for unequal intervals. [4]

4. (a) Compute $\frac{df}{dx}$ and $\frac{d^2f}{dx^2}$ at $x = 3.5$ for the set of values given in $4(b)$. Also use all three point and five point formula to calculate $\frac{df}{dx}$ and $\frac{d^2f}{dx^2}$ at $x = 3$. [8]

   (b) Find the values of $f'(2)$ using Richardson's extrapolation method where $f(x) = x + e^x$ and $h = 0.4$. [6]

5. (a) Derive Simpson's $\frac{1}{3}$ rule to evaluate $\int_a^b f(x)dx$. Use Simpson's $\frac{1}{3}$ rule and Weddle's rule to evaluate $I = \int_{0.2}^{1.4} (\sin x - \ln x + e^x)\, dx$ [7]

   (b) Derive Romberg integration formula to evaluate $\int_a^b f(x)dx$. [7]

6\. (a) Consider, a linear system $Ax = b$ where, [8]

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ and } b = \begin{bmatrix} -1 \\ 7 \\ -7 \end{bmatrix}$$

(i) Check that the Successive Over Relaxation (SOR) method with $w = 1.25$ of the relaxation parameter can be used to solve this system.
(ii) Solve the above system by SOR method starting at the point, $x^{(0)} = (0, 0, 0)^t$ up to three decimal places.

(b) Explain Euler's method and Modified-Euler method to solve the ordinary differential equation: [6]

$$\frac{dy}{dx} = f(x, y), \qquad y(x_0) = y_0.$$

7\. (a) Given the initial value problem: [7]

$$\frac{dy}{dx} = y(1 + x^2), \qquad y(0) = 1$$

find the values of $y(0.2)$ by using the Euler and Modified-Euler methods.

(b) Use the Runge-Kutta 4-th order method to solve: [7]

$$10\frac{dy}{dx} = x^2 + y^2, \qquad y(0) = 1$$

for the interval $0 \leq x \leq 0.4$ with step size $h = 0.2$.

8\. (a) Use Gauss-Seidel and Gauss-Jacobi method to solve the following system of equations: [7]

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3.$$

(b) Solve the following system of equations by LU Decomposition method: [7]

$$5x_1 - 2x_2 + x_3 = 4$$
$$7x_1 + x_2 - 5x_3 = 8$$
$$3x_1 + 7x_2 + 4x_3 = 10.$$

Kishoreganj University

2^{nd} Year 2^{nd} Semester B.Sc. (Hons.) Final Examination-2024
Department of Mathematics
MAT 2207: Discrete Mathematics (3 Credits)

Time: 03.00 Hours **Full Marks: 70**

N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define implication. Explain, without using a truth table, why [5]

$$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$$

is true when p, q, and r have the same truth value and it is false otherwise.

(b) Define tautology and contradiction. Check wether the statement $(p \wedge q) \rightarrow (p \vee q)$ is a tautology or a contradiction. [5]

(c) Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent. [4]

2. (a) What is proof by contraposition? Prove that if n = ab,where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$. [5]

(b) What is argument? Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed." [5]

(c) What is premises? Show that the premises "Everyone in this discrete mathematics class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science." [4]

3. (a) Define Boolean function.Find the sum-of-products expansion for the function $F(x, y, z) = (x + y)\bar{z}$. [4]

(b) Contract the circuits that produce the following outputs: [5]
(i) $\overline{x}(\overline{y + \overline{z}})$ and
(ii) $(x + y + z)(\overline{x}\ \overline{y}\ \overline{z})$.

(c) Draw the Karnaugh maps of these sum-of-products expansions in three variables. [5]
(i) $x\overline{y}\ \overline{z}$ (ii) $\overline{x}yz + \overline{x}\ \overline{y}\ \overline{z}$

4. (a) Why is mathematical induction a valid proof technique? [4]

(b) What is strong induction? Show that whenever $n \geq 3$, $f_n > \alpha^{n-2}$, where $\alpha = \frac{1+\sqrt{5}}{2}$. [5]

(c) Use mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every nonnegative integer n. [5]

5. (a) What is the significance of solution of a recurrence relation? [2]

(b) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$. [5]

(c) Find the solution to the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with initial conditions $a_0 = 1$, $a_1 = -2$, and $a_2 = -1$. [7]

6. (a) What is the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has roots 1,1,1,1,-2,-2,-2,3,3,-4? [3]

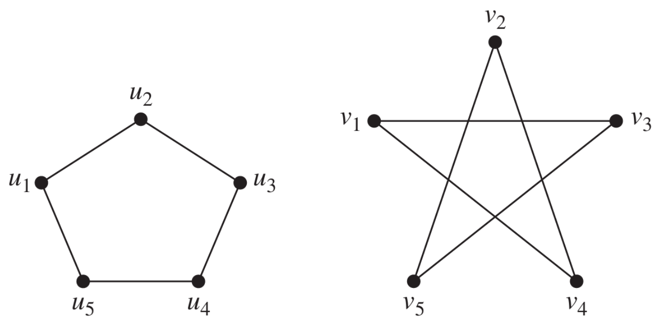
(b) Find all solutions of the recurrence relations [5+6]

i. $a_n = 2a_{n-1} + 3^n$

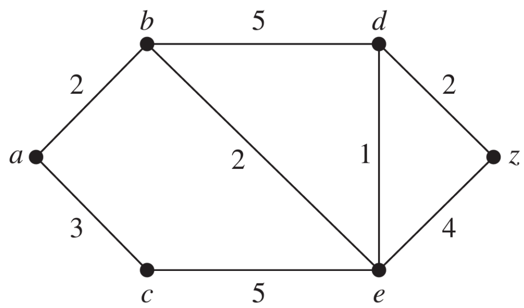
ii. $a_n = 5a_{n-1} - 6a_{n-2} + 2^n + 3n$.

7. (a) What is complete graphs, cycles, wheels and bipartite graph explain with example. [4]

(b) Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists. [5]



(c) Using Dijkstra's algorithm find the length of a shortest path between a and z in the given weighted graph. [5]



8. (a) Find a spanning tree of the simple graph G shown in the following figure. [4]

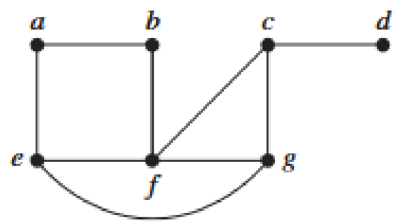


Figure 1: The simple graph G

(b) Define connected and disconnected graph. Show that there is a small path between every pair of distinct vertices of of a connected undirected graph. [6]

(c) Show that neither graph displayed in the following figures has a Hamilton circuit. [4]

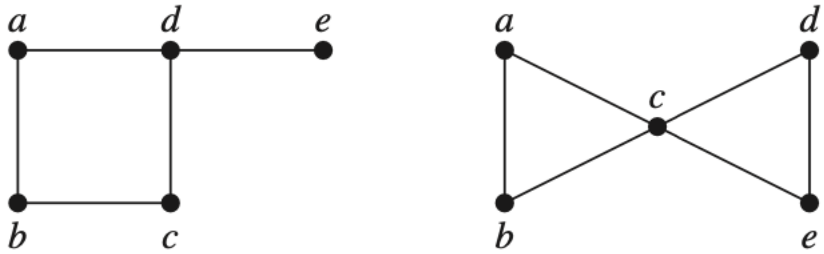


Figure 2: The graph G and H.