# Kishoreganj University

**3rd Year 2nd Semester B.Sc (Hons.) Final Examination-2024**
**Department of Mathematics**
**MAT 3201: Abstract Algebra (3 Credits)**

**Time: 03.00 Hours** **Full Marks: 70**

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**Answer any five questions from the eight given. The numbers in the right margin indicate the full marks.**

1. (a) What is group? Show that the set $\boldsymbol{G = \{0, 1, 2, 3\}}$ is a finite abelian group of order four with respect to addition modulo 4. [6]
   (b) Let $\boldsymbol{G}$ be a group, then show that $\boldsymbol{(ab)^{-1} = b^{-1}a^{-1}}\ \forall \boldsymbol{a, b \in G}$. [4]
   (c) If $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two arbitrary elements of a group $\boldsymbol{G}$, then $\boldsymbol{o(a) = o(b^{-1}ab)}$ [4]

2. (a) Explain permutation with example. [3]
   (b) Let $\boldsymbol{S}$ be a finite set containing $\boldsymbol{n}$ distinct elements. Then, the symmetric set of all permutations of degree $\boldsymbol{n}$ on $\boldsymbol{S}$, forms a finite group of order $\boldsymbol{n!}$ with respect to composite of permutations as the composition. [5]
   (c) State and prove Cayley's theorem. [6]

3. (a) If $\boldsymbol{G}$ is a finite group, show that there exists a positive integer $\boldsymbol{n}$ such that, $\boldsymbol{a^n = e}$ for all $\boldsymbol{a \in G}$. [4]
   (b) Show that, every subgroup of a cyclic group is cyclic. [6]
   (c) Define cosets of a subgroup in a group. Let $\boldsymbol{G = S_3 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6\}}$ be a symmetric group and $\boldsymbol{H = \{\sigma_1, \sigma_4, \sigma_5\}}$ be a subgroup of $\boldsymbol{G}$. Find the cosets of $\boldsymbol{H}$ in $\boldsymbol{G}$? [4]

4. (a) Define normal subgroup of a group. Show that, a subgroup of index $\boldsymbol{2}$ is always a normal subgroup. [4]
   (b) Show that the set $\boldsymbol{G}$ of all matrices of the form $\begin{pmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \\ \boldsymbol{0} & \boldsymbol{a_{22}} \end{pmatrix}$ with real entries such that $\boldsymbol{a_{11}a_{22} \neq 0}$, forms a group under multiplication of matrices, and that the set $\boldsymbol{N}$ of all matrices of the form $\begin{pmatrix} \boldsymbol{1} & \boldsymbol{b} \\ \boldsymbol{0} & \boldsymbol{1} \end{pmatrix}$ is a normal subgroup of $\boldsymbol{G}$. Is $\boldsymbol{N}$ a normal subgroup of $\boldsymbol{GL_2}(\mathbb{R})$? [6]
   (c) If $\boldsymbol{A}$ is a subgroup and $\boldsymbol{N}$ is a normal subgroup of a group $\boldsymbol{G}$, then $\boldsymbol{A \cap N}$ is a normal subgroup of $\boldsymbol{A}$. [4]

5. (a) Suppose $\boldsymbol{G}$ is a group and $\boldsymbol{N}$ is a normal subgroup of $\boldsymbol{G}$. Prove that, the set $\boldsymbol{\frac{G}{N} = \{gN : g \in G\}}$ forms a group under coset operation $\boldsymbol{(aN)(bN) = abN}$. [5]
   (b) If $\boldsymbol{\frac{G}{Z(G)}}$ is cyclic, show that G is abelian. [4]
   (c) Prove that every group $\boldsymbol{G}$ with $|\boldsymbol{G}| < \boldsymbol{6}$ is abelian. [5]

6. (a) What is commutative ring? Prove that (Z,+,•) is a commutative ring. [5]
   (b) Prove that a ring is without zero divisors iff the cancellation laws hold in it. [4]
   (c) If each elements of a ring R is idempotent, show that R must be a commutative ring. [5]

7. (a) Define an integral domain with an example. Prove that, a finite integral domain is a field. [6]

   (b) Prove that, every field is an integral domain. But the converse may not be true. [4]

   (c) Let $\boldsymbol{R}$ be a commutative ring with $\mathbf{1}$ and $\boldsymbol{I}$ be any proper ideal in $\boldsymbol{R}$. Prove that, the ideal $\boldsymbol{I}$ is prime iff the factor ring $\boldsymbol{R}/\boldsymbol{I}$ is an integral domain. [4]

8. (a) Define Kernel of a ring homomorphism. Prove that the kernel of a homomorphism of a ring is a two-sided ideal. [4]

   (b) Prove that every homomorphic image of a ring is isomorphic to some quotient ring. [5]

   (c) Let $\boldsymbol{R}$ be a commutative ring with identity and $\boldsymbol{M}$ an ideal in $\boldsymbol{R}$. Then prove that $\boldsymbol{M}$ is a maximal ideal if and only if $\boldsymbol{R}/\boldsymbol{M}$ is a field. [5]

# Kishoreganj University

$3^{rd}$ Year $2^{nd}$ Semester B.Sc (Hons.) Final Examination-2024
Department of Mathematics
MAT 3202: MATLAB (3 Credits)

**Time: 03.00 Hours** **Full Marks: 70**

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**Answer any five questions from the eight given. The numbers in the right margin indicate the full marks.**

1. (a) Write a MATLAB program to compute the series [5]

$$S_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k}$$

for $n = 10, 50, 100, 500$. Compute the absolute error with respect to $\ln(2)$ and comment on convergence.

(b) Write a MATLAB program to find all prime numbers between $1$ and $200$ using loops and nested `if` conditions. Also determine the total number of prime numbers. [5]

2. (a) Given the matrix [3]

$$A = \begin{bmatrix} -2 & 3 & 0 \\ 4 & -1 & 5 \\ -6 & 2 & -3 \end{bmatrix},$$

write a MATLAB program to replace all negative elements by zero (without using logical indexing) and compute the sum of each row.

(b) Write a MATLAB program to generate a $6 \times 6$ matrix such that diagonal elements are 1, upper triangular elements are 2, and lower triangular elements are 0. [3]

(c) Write a MATLAB program to evaluate the piecewise function [4]

$$f(x) = \begin{cases} x^2 + 1, & x < 0, \\ \sin(x), & 0 \le x \le \pi, \\ \ln(x), & x > \pi, \end{cases}$$

for $x = -2 : 0.2 : 4$ and plot the function.

3. Consider the function $f(x) = 6x^4 - 12x^3 + 8x^2 - 4x$. Sketch the graph of $f(x), f'(x)$ and $f''(x)$ in the same diagram. Use these graphs to [10]

   i. Find the intervals of increase or decrease.
   ii. Find the local maximum and minimum values.
   iii. Find the intervals of concavity and inflation points.

Also draw the local maximum, minimum and inflation points on the graph. (Use different style)

4. (a) For the augmented matrix [3]

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & 4 & 6 & 12 \\ 1 & 1 & 1 & 4 \end{array}\right],$$

determine using MATLAB whether the system is consistent.

(b) Plot the function [4]

$$f(x) = xe^{-x}, \quad 0 \leq x \leq 5,$$

find its critical point, and verify the maximum graphically.

(c) Plot the parametric curve [3]

$$x(t) = \cos(t), \quad y(t) = \sin(2t), \quad 0 \leq t \leq 2\pi,$$

and comment on its shape.

5. (a) Solve the equation $cos(x) - xe^{x} = 0$ using bisection method correct up to 3 decimal places. [5]

(b) Using Newtons interpolation formula find the values of F(0.375) where [5]

Table 1:

| x | 0.0 | 1.20 | 1.40 | 1.60 | 1.80 | 2.00 |
|---|---|---|---|---|---|---|
| F(x) | 0.242 | 0.1942 | 0.1497 | 0.1109 | 0.079 | 0.054 |

6. (a) Evaluate the definite integral $\int_0^2 \frac{1}{x^2+4}\, dx$ using [10]

i. Simpson's 3/8 rule,

ii. Romberg integration.

# Kishoreganj University

**$3^{rd}$ Year $2^{nd}$ Semester B.Sc (Hons.) Final Examination-2024**
**Department of Mathematics**
**MAT 3203: Theory of Numbers (3 Credits)**

**Time: 03.00 Hours** **Full Marks: 70**

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**Answer any five questions from the eight given. The numbers in the right margin indicate the full marks.**

1. (a) State and prove the division algorithm. [6]

   (b) Discuss Euclidean algorithm and using this algorithm find the GCD of the integers $\mathbf{172}$ and $\mathbf{20}$. [4]

   (c) Let $\boldsymbol{a}$ and $\boldsymbol{b}$ are integers not both zero, then prove that there exists integers $\boldsymbol{x}$ and $\boldsymbol{y}$ such that $\boldsymbol{gcd(a,b) = ax + by.}$ [4]

2. (a) Define Diophantine equation. Using the idea of linear Diophantine equation solve the problem, [4]

$$\mathbf{18x + 5y = 48}$$

   (b) Prove that the linear Diophantine equation $\boldsymbol{ax + by = c}$ has a solution if and only if $\boldsymbol{d|c}$ where $\boldsymbol{d = gcd(a,b).}$ Also Prove that, if $\boldsymbol{x_0, y_0}$ be any particular solution of $\boldsymbol{ax + by = c}$ then all other solutions are given by $\boldsymbol{x = x_0 + (\frac{b}{d})t, \quad y = y_0 - (\frac{a}{d})t,}$ [6]

   (c) If $\boldsymbol{ac \equiv bc \pmod{n}}$, then show that $\boldsymbol{a \equiv b \pmod{n/d}}$ where $\boldsymbol{gcd(c,n) = d.}$ [4]

3. (a) Solve inconsistency of the system [5]

$$\boldsymbol{x \equiv 1 \pmod{6}, \quad x \equiv 3 \pmod{10}, \quad x \equiv 5 \pmod{15}.}$$

   (b) Prove the Chinese Remainder Theorem for the case when the moduli are not pairwise coprime. [5]

   (c) Solve the simultaneous congruences [4]

$$\boldsymbol{2x + y \equiv 1 \pmod{11}, \quad x - 3y \equiv 4 \pmod{11}.}$$

4. (a) Find the continued fraction expansion of [5]

$$\boldsymbol{\frac{1 + \sqrt{13}}{2}}$$

   and determine its period.

   (b) Let $\boldsymbol{\frac{p_n}{q_n}}$ be the $\boldsymbol{n}$-th convergent of $\boldsymbol{\alpha}$. Prove that [5]

$$\boldsymbol{\left|\alpha - \frac{p_n}{q_n}\right| < \frac{1}{q_n q_{n+1}}.}$$

   (c) Define a Euclidean quadratic field and prove that $\mathbb{Q}(\sqrt{-1})$ is Euclidean. [4]

5. (a) Let $\boldsymbol{a_0, a_1, a_2, \ldots}$ be an infinite sequence of integers with $\boldsymbol{a_1, a_2, \ldots}$ positive, and let $\boldsymbol{C_k = [a_0; a_1, a_2, \ldots, a_k]}$. Then prove that the convergents $\boldsymbol{C_k}$ tend to a limit $\boldsymbol{\alpha}$, i.e [8]

$$\boldsymbol{\lim_{k\to\infty} C_k = \alpha.}$$

(b) Using Fermat's factorization method, factor the following positive integers: [6]
(i)**7709** (ii) **11021**

6. (a) Using congruences, determine the last three digits of $\mathbf{7^{2026}}$. [4]

(b) Prove that in a round-robin tournament with an odd number of teams, each team plays exactly once in each round. [5]

(c) Derive the ISBN-10 check digit formula and prove that it detects all single-digit errors and all transposition errors except those involving **0** and **9**. [5]

7. (a) Define quadratic residue and nonresidue. Find all the quadratic residues of **3** and **13**. [6]

(b) State and prove the Euler's Criterion for an odd prime $\boldsymbol{p}$ and an integer $\boldsymbol{a}$ not divisible by $\boldsymbol{p}$. [4]

(c) Find all solutions of the quadratic congruence [4]

$$\boldsymbol{x^2 + x + 1 \equiv 0 \pmod{7}}$$

8. (a) Decipher the ciphertext message [4]

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LFDPH LVDZL FRQTX HUHG
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that has been enciphered using the Caesar cipher.

(b) Using the digraphic cipher that sends the plaintext block $\boldsymbol{P_1P_2}$ to the ciphertext block $\boldsymbol{C_1C_2}$ with [6]

$$\boldsymbol{C_1 \equiv 3P_1 + 10P_2 \pmod{26}}$$
$$\boldsymbol{C_2 \equiv 9P_1 + 7P_2 \pmod{26},}$$

encipher the message.

(c) A government agency is designing a secure document distribution system. They need to choose between using digital signatures and Message Authentication Codes (MACs) for authenticating official announcements sent to thousands of citizens. Explain in detail the fundamental differences between a digital signature and a Message Authentication Code. [4]

# Kishoreganj University

**3rd Year 2nd Semester B.Sc (Hons.) Final Examination-2024**

**Department of Mathematics**

**MAT 3205: Numerical Analysis-II (3 Credits)**

**Time: 03.00 Hours** **Full Marks: 70**

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**Answer any five questions from the eight given. The numbers in the right margin indicate the full marks.**

1. (a) Find the coefficients $a_0, a_1, a_2$ and $a_3$ of the polynomial $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ that best fits the $n$ data points $(x_i, y_i)$ for $i = 1, 2, \ldots, n$. [7]

   (b) A tension test is conducted for determining the stress-strain behavior of rubber. The data points from the test are given below. Determine the third order polynomial that best fits the data points. [7]

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| strain $\epsilon$ | 0.4 | 1.2 | 2.0 | 2.8 | 3.6 | 4.4 | 5.2 | 6.0 |
| stress $\sigma$ (MPa) | 3.0 | 5.8 | 5.8 | 7.4 | 15.6 | 26.7 | 35.6 | 41.5 |

2. (a) If $A$ is an $n \times n$ diagonalizable matrix with a dominant eigenvalue, then prove that there exists a nonzero vector such that the sequence of vectors given by [7]

$$Ax_0, A^2x_0, A^3x_0, \ldots, A^kx_0, \ldots$$

   approaches a multiple of the dominant eigenvector of $A$.

   (b) Using Inverse Power Method, find the least eigenvalue and the corresponding eigenvector of the following matrix start with the vector $x = [1, 1, 1]^T$. [7]

$$\begin{pmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{pmatrix}$$

3. (a) Define Householder transformation. Prove that if a Householder transformation is symmetric and orthogonal then $P^{-1} = P$. [5]

   (b) Use the Householder reduction to transform the following matrix into a tridiagonal matrix. [9]

$$\begin{pmatrix} 4 & -2 & 1 & -1 \\ -2 & 4 & -2 & 1 \\ 1 & -2 & 4 & -2 \\ -1 & 1 & -2 & 4 \end{pmatrix}$$

4. (a) Discuss the advantages and disadvantages of the Newton's method, the quasi-Newton's method, and the Steepest Descent method. [7]

   (b) Solve the non-linear system [7]

$$\begin{aligned} 3x_1 - \cos(x_2x_3) - \frac{1}{2} &= 0 \\ 4x_1^2 - 625x_2^2 + 2x_2 - 1 &= 0 \\ e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} &= 0 \end{aligned}$$

   by Newton's method with $\underline{x}^{(0)} = \underline{0}$ to compute $\underline{x}^{(1)}$ and $\underline{x}^{(2)}$.

5. (a) Discuss Backward Euler Method to solve the initial value problem [5]

$$\frac{dy}{dt} = f(t, y), \quad a \le t \le b, \quad y(a) = \alpha$$

(b) Explain why, for sufficiently small step sizes $h$, the second-order Taylor method yields a more accurate approximation to the exact solution than the Euler method. [2]

(c) Use Euler method to approximate the solution to [7]

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \quad y(0) = 0.5 \text{ with } h = 0.5$$

and compare approximate solution with the exact values given by

$$y(t) = (t+1)^2 - 0.5e^t$$

6. What do you mean by Predictor-Corrector Method? Given that the Predictor-Corrector formulas, [14]
Predictor: $w_4^{(0)} = w_3 + \frac{h}{24}\left[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)\right]$
Corrector: $w_4^{(1)} = w_3 + \frac{h}{24}\left[9f\left(t_4, w_4^{(0)}\right) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)\right]$
Compute $w(1.8)$ for the IVP $y'(t) = 1 + \frac{y}{t}; \quad 1 \le t \le 2; \quad y(1) = 2$ by applying above Predictor-Corrector formulas using $h = 0.2$. Use starting values obtained from the Runge-Kutta method of order four.

7. (a) Explain briefly non-linear shooting method. Use this technique with $h = 0.25$ to approximate the solution to the boundary value problem (BVP) $y'' = 2y^3$ for $1 \le x \le 2$, where $y(1) = 0.25$ and $y(2) = 0.2$. Compare your results to the actual solution $y(x) = \frac{1}{x+3}$. [7]

(b) Consider a thin metal plate with dimensions $0.5m$ by $0.5m$ . The heat distribution in the plate can be expressed as: $u_{xx} + u_{yy} = 0$ for $(x, y)$ in the set $R = \{(x, y) : 0 < x < 0.5, 0 < y < 0.5\}$ with the boundary conditions $u(0, y) = 0, u(x, 0) = 0, u(x, 0.5) = 200x, u(0.5, y) = 200y$. Calculate the steady-state temperature at the interior point assuming a grid size of $h = k = 0.125m$. Use Gauss-seidel iterative method to compute the values of $u$ at the internal mesh points. [7]

8. Derive the three level explicit finite difference scheme to solve the following BVP: $u_{tt} = u_{xx}, 0 < x < l, t > 0$, subject to the conditions $u(x, 0) = f(x), u_t(x, 0) = g(x), 0 \le x \le l, u(0, t) = \psi_1(t), u(l, t) = \psi_2(t), t > 0$. Approximate the solution to the wave equation at $t = 0.4$; [14]

$$\begin{aligned} &u_{tt} = u_{xx}, \\ &u(0, t) = u(1, t) = 0, t > 0, \\ &u(x, 0) = \sin^3 \pi x; 0 \le x \le 1 \\ &u_t(x, 0) = 0, 0 \le x \le 1 \end{aligned}$$

with $h = 0.25$ and $k = 0.2$.

# Kishoreganj University

## $3^{rd}$ Year $2^{nd}$ Semester B.Sc (Hons.) Final Examination-2024
## Department of Mathematics
## MAT 3207: Mathematical Methods (3 Credits)

**Time: 03.00 Hours** **Full Marks: 70**

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**Answer any five questions from the eight given. The numbers in the right margin indicate the full marks.**

1. (a) Define the Fourier series. Find the Fourier series of the function [6]

$$f(x) = \begin{cases} 0 & \text{when} \quad -2 < x < -1 \\ k & \text{when} \quad -1 < x < 1 \\ 0 & \text{when} \quad 1 < x < 2 \end{cases}.$$

(b) Establish the Fourier series for even function. [4]

(c) Determine Parseval's formula for particular case. [4]

2. (a) Prove that $\mathcal{L}\{\cosh at\} = \dfrac{s}{s^2 - a^2}$ if $s > |a|$. [4]

(b) State the first translation property of Laplace transform. Using this property find [6]

i $\mathcal{L}\{e^{-2t} \sin 4t\}$

ii $\mathcal{L}\{e^{4t} \cosh 5t\}$

(c) If $\mathcal{L}\{F(t)\} = f(s)$, prove that [4]

$$\mathcal{L}\{F'''(t)\} = s^3 f(s) - s^2 F(0) - sF'(0) - F''(0)$$

3. (a) Define Laplace transformation of a function $F(t)$.. Calculate the laplace transform of the function $F(t)$, where [5]

$$F(t) = \begin{cases} t & \text{for} \quad 0 < t < 2 \\ 3 & \text{for} \quad t > 2 \end{cases}$$

(b) State and prove existence theorem for the laplace transform. [5]

(c) Prove that $\displaystyle\int_0^\infty te^{-st} \cos at \, dt = \frac{s^2 - a^2}{(s^2 + a^2)^2}$. [4]

4. (a) Show that $\int_0^\infty e^{-u^2} \, du = \frac{\sqrt{\pi}}{2}$. [4]

(b) Evaluate [4]

i $\displaystyle\int_0^\infty t \, e^{-2t} \cos t \, dt$

ii $\displaystyle\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} \, dt$

(c) State the Convolution theorem and evaluate the following by using this theorem $\mathcal{L}^{-1}\left\{\dfrac{s}{(s^2 + a^2)^2}\right\}$. [6]

5. (a) Find [4]

$$\mathcal{L}^{-1}\left\{\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right\}.$$

(b) Solve the following differential equation by using Laplace Transform [6]

$$Y''' - 3Y'' + 3Y' - Y = t^2e^t, \quad Y(0)=1, \quad Y'(0)=0, \quad Y''(0)=-2.$$

(c) Find the Fourier transform of $f(x)=1$ if $|x|<1$ and $f(x)=0$ otherwise. [4]

6. (a) Find the fourier sine transform of $e^{-x},\ x\geq 0$. [3]

(b) Determine the fourier integral of the function $f(x) = e^{-kx}$ when $x > 0$ and $f(-x)=f(x)$ for $k>0$, and hence prove that [4]

$$\int_0^\infty \frac{\cos ux\ du}{k^2+u^2} = \frac{\pi}{2k}e^{-kx}.$$

(c) Use finite fourier transforms to solve [7]

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2};\ U(0,t)=0,\ U(4,t)=0,$$
$$U(x,0)=2x, \text{ where } 0<x<4,\ t>0,$$

and interpret physically.

7. (a) Show that the set of functions $f_n(x)=\sin nx,\ n\in\mathbb{N}$ is orthogonal on the interval $0<x<\pi$. Also, find the corresponding orthonormal set. [6]

(b) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem [8]

$$y''+\lambda y=0,\ y(0)=0,\ y(\pi)=0.$$

8. Determine the Green's function and hence solve the nonhomogeneous boundary value problem $y''+\lambda y=\cos kt,\ y(0)=y(\pi)=0$, where $k$ is a positive constant. [14]