

Kishoreganj University

Department of Mathematics

1st Year 2nd Semester B.Sc (Hons.) Final Examination-2025

Course Code: MAT 1201 Course Title: Linear Algebra-I

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Prove that, every square matrix can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix. [4]

- (b) If A and B are comparable matrices and A^T and B^T are the transpose matrices of A and B respectively, then show that [4]

$$(i) \quad (A + B)^T = A^T + B^T,$$

$$(ii) \quad (AB)^T = B^T A^T.$$

- (c) For what values of μ and α , the following linear equations have a solution and solve them completely in each case: [6]

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \alpha z = \mu$$

2. (a) Define rank of a matrix. Reduce the matrix A to the normal form and hence obtain its rank where [7]

$$A = \begin{bmatrix} 2 & 6 & -5 & 8 \\ 4 & 3 & -1 & 7 \\ -1 & 1 & 6 & 0 \\ 5 & 1 & 4 & 7 \end{bmatrix}$$

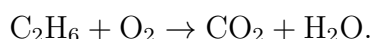
- (b) Find the inverse of the following matrix by using row canonical form: [7]

$$A = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 3 & 0 & 2 & 2 \\ 2 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

3. (a) Explain the method of polynomial interpolation using linear systems. Derive the linear system corresponding to a polynomial passing through a given set of points. [4]

- (b) Find the quadratic polynomial $p(x) = ax^2 + bx + c$ that passes through the points $(1, 2)$, $(2, 3)$, $(3, 8)$, by formulating and solving a system of linear equations. [5]

- (c) Balance the following chemical equation using a system of linear equations: [5]



4. (a) Define a vector space. Consider the set $V = \left\{ \begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix} \in M_2(\mathbb{R}) \mid a, b \in \mathbb{R} \right\}$. Test whether V is a vector space or not. If not, then list all axioms that fail to hold. [4]

- (b) Define a subspace of a vector space with an example. [5]

Let $V = \mathbf{R}^3$. Show that W is not a subspace of V , where (i) $W = \{(a, b, c) : a \geq 0\}$, (ii) $W = \{(a, b, c) : a^2 + b^2 + c^2 \leq 1\}$.

- (c) Define linear combination. If $S = \{\underline{v}_1, \underline{v}_2, \dots, \underline{v}_n\}$ is a set of vectors in a vector space V , then prove that the set $W = L(S)$ of all linear combinations of vectors in S is a subspace of V . [5]

5. (a) Define basis and dimension of a vector space. Prove that the vectors $(1, 2, 0)$, $(0, 5, 7)$ and $(-1, 1, 3)$ form a basis for $\mathbb{R}^3$. [6]

- (b) Determine whether the vectors $v_1 = (2, -1, 3)$, $v_2 = (4, 1, 2)$ and $v_3 = (8, -1, 8)$ generate (span) $\mathbb{R}^3$. [4]

- (c) Define linear transformation. In $\mathbb{R}^2$ consider the function $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by $T(x, y) = (x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$ where θ is a fixed number and $0 \leq \theta \leq 2\pi$. Then T is a linear transformation. [4]

6. (a) Consider the basis $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$, where $\mathbf{v}_1 = (1, 1, 1)$, $\mathbf{v}_2 = (1, 1, 0)$, and $\mathbf{v}_3 = (1, 0, 0)$. Take into account $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation such that [7]

$$T(\mathbf{v}_1) = (1, 0), \quad T(\mathbf{v}_2) = (2, -1), \quad T(\mathbf{v}_3) = (4, 3).$$

Find a formula for $T(x_1, x_2, x_3)$; then use this formula to compute $T(2, -3, 5)$.

- (b) Define rank and nullity. Find rank and nullity of the matrix [7]

$$A = \begin{pmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{pmatrix}.$$

Hence verify that $\text{rank}(A) + \text{nullity}(A) = \text{number of columns of } A$.

7. (a) Define kernel and range of a linear transformation. If $T : U \rightarrow V$ is a linear transformation, then [6]

- (i) the kernel of T is a subspace of U ,
(ii) the range of T is a subspace of V .

- (b) Find the eigenvalues and eigenvectors of the matrix [8]

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

Also find the matrix P that diagonalizes A and determine $P^{-1}AP$.

8. (a) State and prove Cayley-Hamilton theorem. [7]

- (b) Using Cayley-Hamilton theorem find the inverse of the matrix [7]

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Kishoreganj University

Department of Mathematics

1st Year 2nd Semester B.Sc (Hons.) Final Examination-2025

Course Code: MAT 1203 Course Title: Integral Calculus

Total Marks: 70 Time: 03.00 Hours Credits: 3

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Let A denote the area between the graph of $f(x) = \sqrt{x}$ and the interval $[0, 1]$, and let B denote the area between the graph of $f(x) = x^2$ and the interval $[0, 1]$. Explain geometrically why $A + B = 1$. [4]

- (b) Find a function f such that the slope of the tangent line at a point (x, y) on the curve $y = f(x)$ is $\sqrt{3x + 1}$ and the curve passes through the point $(0, 1)$. [4]

- (c) Evaluate (i) $\int [\csc^2 t - \sec t \tan t] dt$, (ii) $\int \frac{y}{\sqrt{2y+1}} dy$, (iii) $\int_{-2}^2 (1 - 3|x|) dx$. [6]

2. (a) Use the Fundamental Theorem of Calculus, to find the value of [6]

$$(i) \frac{d}{dx} \left[\int_x^\pi \cos(\sqrt{t}) dt \right] \quad (ii) \frac{d}{dx} \left[\int_0^x e^{t^2} dt \right]$$

- (b) State Walli's formula. If $U_n = \int_0^1 x^n \arctan x dx$, then prove that [8]

$$(n+1)U_n + (n-1)U_{n-2} = \frac{\pi}{2} - \frac{1}{n}.$$

3. (a) State the Mean-Value Theorem for Integrals. Verify the function $f(x) = x^2 + x$ is continuous on the interval $[-12, 0]$ by the Mean-Value Theorem for Integrals. [5]

- (b) Suppose that a particle moves along a coordinate line with constant acceleration. Show that the average velocity of the particle during a time interval $[a, b]$ matches the velocity of the particle at the midpoint of the interval. [4]

- (c) Take into account [5]

$$I = \int_0^a \frac{f(x)}{f(x) + f(a-x)} dx.$$

Show that $I = a/2$.

[Hint: Let $u = a - x$, and then note the difference between the resulting integrand and 1.]

4. (a) Find the volume of the solid generated when the region under the curve $y = x^2$ over the interval $[0, 2]$ is rotated about the line $y = -1$. [4]

- (b) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y -axis. [4]

- (c) A rocket weighing 3 tons is filled with 40 tons of liquid fuel. In the initial part of the flight, fuel is burned off at a constant rate of 2 tons per 1000ft of vertical height. How much work in foot-tons (ft·ton) is done lifting the rocket 3000 ft? [4]

- (d) Find the arc length of the curve $y = x^{3/2}$ from $(1, 1)$ to $(2, 2\sqrt{2})$. [2]

5. (a) What do you mean by the discontinuity at an Interior Point? Evaluate $\int_0^1 \frac{1}{\sqrt{x}} dx$. [4]
 (b) Test the convergence of $\sum_{n=1}^{\infty} \frac{5^n}{n!}$. [4]
 (c) Applying the Concept to an Integral Determine if $\int_1^{\infty} \frac{1}{\sqrt{x^3+1}} dx$ converges. [6]
6. (a) A bag contains 10 balls of which 4 are black. If 3 balls are selected at random without replacement, obtain the probability distribution for the number of black balls drawn. [4]
 (b) Prove that $\Gamma(n+1) = n! = n\Gamma(n)$. [4]
 (c) Determine (i) $\Gamma\left(\frac{7}{2}\right)$, (ii) $\Gamma\left(-\frac{3}{2}\right)$. [2]
 (d) Show that [4]

$$\beta(m, n) = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}} dy, \quad \text{when } m, n > 0.$$

7. (a) Express the polar equation [6]

$$r = 2 + \cos\left(\frac{5\theta}{2}\right)$$

parametrically, and generate the polar graph from the parametric equations using a graphing utility.

- (b) Find the area of the region that lies inside the circle $r = 3 \sin \theta$ and outside the cardioid $r = 1 + \sin \theta$. [4]
 (c) The curve $y = \sqrt{4-x^2}$, $-1 \leq x \leq 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about the x -axis. [4]
8. (a) Find the length of the arc of the parabola $y^2 = 4ax$ between the vertex and arc end of the latus rectum. [4]
 (b) Determine the area of the region enclosed by the rose curve $r = \cos 2\theta$. [4]
 (c) Evaluate the volume and the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ about its base. [6]

Kishoreganj University

1st Year 2nd Semester B.Sc. (Hons.) Final Examination 2025

Department of Mathematics

STA 1205: Introduction to Probability (3 Credits)

Time: 03:00 Hours

Full Marks: 70

**N.B.: Answer any FIVE (5) questions from the following EIGHT (8) questions.
Numbers given in the right margin indicate the marks of the respective questions.**

1. (a) Define Discrete and Continuous Sample Space, Mutually Exclusive Events. [3]
(b) In an office of **100** employees, **75** read English, **50** read Bangla dailies and **40** read both. An employee is selected at random. What is the probability that the selected employee
(i) Reads English newspaper? (ii) Reads at least one of the papers? (iii) Reads none?
(iv) Reads English but not Bangla? [5]
(c) The probability that a married man watches a certain TV show is **0.4** and that his wife watches the show is **0.5**. The probability that a man watches the show, given that his wife does, is **0.7**. Find [6]
(i) The probability that a married couple watches the show.
(ii) The probability that a wife watches the show given that her husband does.
(iii) The probability that at least one of the partners will watch the show.
2. (a) Define the probability mass function and probability density function. [4]
(b) Verify if the following functions are probability mass functions. [5]
(i) $f(x) = \frac{2x-1}{8}$, $x = 0, 1, 2, 3$. (ii) $g(x) = \frac{x+1}{16}$, $x = 0, 1, 2, 3$.
(c) A random variable X has the following functional form: [5]

$$f(x) = kx, \quad 0 < x < 4 \\ = 0, \quad \text{otherwise.}$$

- (i) Determine k for which $f(x)$ is a density function.
(ii) Find $P(1 < X < 2)$ and $P(X > 2)$.
3. (a) State and prove Bayes' theorem [7]
(b) A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelop just two consecutive letters TA are visible. What is the probability that the letter came from CALCUTTA? [7]
4. (a) For two continuous random variable X and Y , the joint density is [4]

$$f(x, y) = 2xe^{-y}; 0 \leq x \leq 1, 0 \leq y < \infty.$$

Determine marginal densities of X and Y and hence examine whether X and Y are independent.

- (b) Consider the following probability distribution of X and Y . [5]

$$f(x, y) = \frac{x+y}{21}, x = 1, 2, 3 \text{ and } y = 1, 2.$$

- (i) Obtain the marginal and conditional distributions of X and Y . (ii) Find $f(x | 1)$ and
(iii) $P(X = 2 | Y = 1)$.

- (c) Suppose $\mathbf{X}$ and $\mathbf{Y}$ have the following joint probability function [5]

$\mathbf{Y}$	$\mathbf{X}$		
	2	4	Row sum
1	0.10	0.15	0.25
3	0.20	0.30	0.50
5	0.10	0.15	0.25
Column sum	0.40	0.60	1.00

Check if $\mathbf{X}$ and $\mathbf{Y}$ are independent.

5. (a) Show that mean and median of normal distribution is equal. [6]
 (b) Find the moment generating function of normal distribution about origin. [8]
6. (a) Define Binomial probability distribution with example. [6]
 Drive the first four raw moments from MGF of the Binomial probability distribution.
 (b) Given the following joint density function of the random variables $\mathbf{X}$ and $\mathbf{Y}$: [8]

$$f(x, y) = \frac{x(3y^2 + 1)}{4}, \quad 0 < x < 2, 0 < y < 1$$

$$= 0, \quad \text{elsewhere}$$

- (i) Check that $E(\mathbf{X} + \mathbf{Y}) = E(\mathbf{X}) + E(\mathbf{Y})$.
 (ii) Find $E\left(\frac{\mathbf{Y}}{\mathbf{X}}\right)$. And then verify if $E\left(\frac{\mathbf{Y}}{\mathbf{X}}\right) = \frac{E(\mathbf{Y})}{E(\mathbf{X})}$
7. (a) Define Poisson distribution with example. Give some properties of Poisson distribution. [4]
 (b) Suppose, $\mathbf{X}$ is a discrete random variable having the following probability function: [5]

$$f(x) = \frac{e^{-m} m^x}{x!}, \quad x = 0, 1, 2, \dots$$

Find the moment generating function of $\mathbf{X}$ and its mean and variance.

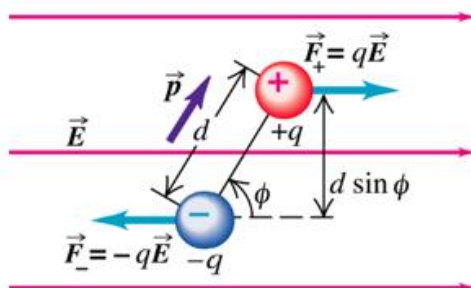
- (c) The average number of emergency patients in a given day in a private clinic is **10** . Find the probability that in a specified day, the clinic will receive (i) **5** emergency patients (ii) at least **3** emergency patients (iii) between **5** and **10** emergency patients, if the number of patients is assumed to follow the Poisson distribution? [5]
8. (a) A and B play a game in which their chances of wining are in the ratio 3:2. Find A's chance of winning at least three games out of the five games played. [5]
 (b) Find the moments of the Poisson distribution. [9]

Kishoreganj University
1st Year 2nd Semester B.Sc. (Hons.) Final Examination 2025
Department of Mathematics
PHY 1207: Electricity & Magnetism (3 Credits)
Full Marks: 70, Time: 3 hours

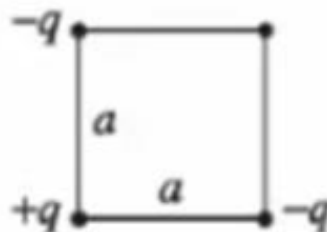
Answer any five (5) out of eight (8) questions

(Figures shown in the right margin indicate full marks)

1. a) What is Coulomb's law? Describe the components of the associated equation, and how is it similar to Newton's law of gravity? (4)
- b) Derive an expression for the electric field at any point on the equatorial line (perpendicular bisector) of an electric dipole. (6)
- c) An electric dipole is in uniform electric field of magnitude of $E = 2 \times 10^4 \text{ N/C}$ that is directed parallel to the plane as shown in the figure. The charges: $q = \pm 3 \times 10^{-19} \text{ C}$ and both charges are laying in the plane and separated by a distance of $d = 0.4 \text{ nm}$. Find the (i) dipole moment (p), (ii) torque (τ), and (iii) total external force ($F_{\pm}$). Take $\phi = 30^\circ$. (4)



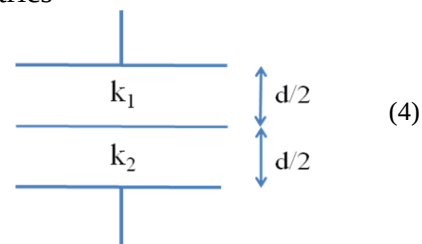
2. a) Define an equipotential surface. Why is no work done when a charge moves along an equipotential surface? (4)
- b) A system consists of N stationary point charges located at different positions in space. Starting from the definition of electric potential due to a single point charge, derive an expression for the electric potential at an arbitrary point due to a system of N point charges using the principle of superposition. (6)
- c) Three point charges are placed at three corners of a square of side a , as shown in the figure. A fourth charge $+q$ is brought from infinity and placed at the empty (top-right) corner of the square. Determine the change in the electrostatic potential energy of the system due to the addition of this charge. (4)



3. a) What is a capacitor and how do you define the capacitance of the capacitor? (3)
- b) Show that for a spherical capacitor $C = 4\pi\epsilon_0 \frac{ab}{b-a}$, where the symbols carry usual meanings. Hence what would be the value of capacitance for an isolated sphere? (5+2)

- c) A parallel plate capacitor of plate area A is filled with two dielectrics as shown in the figure. Show that the capacitance of the capacitor is,

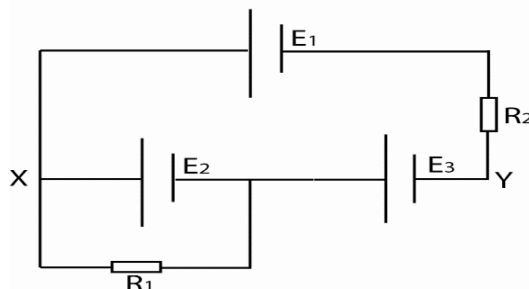
$$C = \left(\frac{2\epsilon_0 A}{d} \right) \frac{k_1 k_2}{k_1 + k_2}.$$



4. a) What is drift speed? Show that the expression of drift speed of electrons through a conducting wire is given by $\vec{v}_d = \frac{\vec{J}}{ne}$, where the symbols have their usual meanings. (2+4)

- b) State and explain Kirchoff's current and voltage rules. (4)

- c) Find the current in each resistor and potential difference between X and Y. Given; $E_1=6.0$ V, $E_2=5.0$ V, $E_3=4.0$ V, $R_1=100 \Omega$, $R_2=50 \Omega$. (4)



5. a) What is time constant of an RC circuit? What is its unit? (2+2)

- b) Using Kirchoff's law, derive the differential equation for discharging of a capacitor in RC circuit and find the expression of discharging current. Also find the expression of voltage across the resistor and capacitor in the circuit during discharging. (6)

- c) A RC circuit has $R = 1 k\Omega$ and $C = 10 \mu F$ connected in series. How long it will take to reduce half of its initial charge? (4)

6. a) State the Biot-Savart law and explain briefly with aid of a sketch. (4)

- b) Use Biot-Savart law to show that the magnetic field strength B at a distance x along the axis of a thin coil, radius a , of N turns of wire carrying a current I is given by (6)

$$B = \frac{\mu_0 I}{2} \frac{Na^2}{(a^2 + x^2)^{3/2}}.$$

- c) In the Bohr model of hydrogen atom, the electron circulates around the nucleus in a path of radius 5×10^{-11} meter at a frequency 6.8×10^{15} rev/sec. (4)

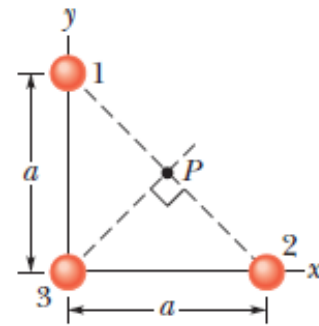
- (i) What is the magnetic field strength B is set up at the center of the orbit?
(ii) What is the equivalent magnetic dipole moment?

7. a) Define flux of an electric field and explain how Gauss's law relates the electric flux with charges. (3)

- b) Show that the electric field strength E at a distance r from the center of a sphere with a charge q distributed uniformly and of radius R is given by $E = \left(\frac{q}{4\pi\epsilon_0 R^3} \right) r$. Plot the field (6)

strength E as a function of r (for $r = 0$ to $r \gg R$) and explain the points where the value of E will be minimum and maximum.

- c) In Fig., the three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = 2e$. Distance $a = 6.00$ mm. (i) What is the magnitude of the net electric field at point P due to the particles? (ii) If you place an electron at point P , what acceleration the electron would experience and in which direction?



(5)

8. a) What is a transformer?

(2)

b) Discuss the construction and theory of a transformer and show that for an ideal

(5+3)

transformer, $\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$, where the symbols have their usual meanings.

c) Using the theory discussed in question 6. (b), explain how to design a step-up and step-down transformer.

(4)

Kishoreganj University

1st Year 2nd Semester B.Sc (Hons.) Final Examination-2025

Department of Mathematics

MAT 1202: MATHEMATICA Lab (3 Credits)

Time: 03.00 Hours

Full Marks: 60

Answer all the questions. The numbers in the right margin indicate the full marks.

1. (a) Marks obtained by a school students in the final examination in different courses are given in the following table: [5]

Courses	Bangla	English	Mathematics	Science	ICT	Religion
Marks Obtained	79	55	91	45	32	81

Write a MATHEMATICA program to calculate the grade point (GP) for each course and then calculate the grade point average (GPA) using the following grading system:

Marks	80–100	70–79	60–69	50–59	40–49	33–39	0–33
Grade Point	5.00	4.00	3.50	3.00	2.00	1.00	0.00

- (b) Compute the sum and the product of the following sequence terms: [4]

$$1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{3}, \dots, 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{20}$$

- (c) Input a number n from the keyboard and accomplish the following steps:

i. Check whether the number n is a palindrome or not. [3]

ii. Print the first n prime numbers. [3]

2. (a) Let $f(x) = \begin{cases} \frac{1}{(x-2)}, & x < -2 \\ x^2 - 5, & -2 < x \leq 3. \\ \sqrt{x+13}, & x > 3 \end{cases}$ [8]

(i) Find the limit of $f(x)$ as (i) $x \rightarrow 2$ (ii) $x \rightarrow 0$, and (iii) $x \rightarrow 3$ verify that the limit calculations in parts (i), (ii), and (iii) are consistent with the graph of $f(x)$.

(ii) Examine the continuity and differentiability of the function $f(x)$ at $x = \frac{\pi}{2}$, where

$$f(x) = \begin{cases} 1, & x < 0 \\ 1 + \sin x, & 0 \leq x < \frac{\pi}{2} \\ 2 + (x - \frac{\pi}{2})^2, & x \geq \frac{\pi}{2} \end{cases}$$

- (b) Let $f(x) = x^2 - 8x + 15$. Verify the hypotheses of Rolle's Theorem on a given interval $[3, 5]$, and find all values of $c \in (3, 5)$ that satisfy the conclusion of this theorem. [3]

- (c) (i) Find the entire length of the curve [4]

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1.$$

(ii) Compute the common area to the cardioid $r = a(1 + \cos \theta)$ and the circle $r = \frac{3}{2}a$.

3. (a) The equation $r = \sin n\theta$, where n is a positive integer that represents a family of polar curves. Investigate the behavior of this family and form a conjecture about how the number of loops is related to n . [4]

(b) Find the volume of the solid that is obtained when the region under the curve $y = x^3$ over the interval $[0, 4]$ is revolved about the x -axis. [3]

(c) Consider the parabolic equation $x^2 = 16y$.

i. Find the focus, vertices and directrix of the parabola. [1]

ii. Sketch the parabola, focus, vertices and directrix in same set of axes with different color. [2]

(d) Consider the two matrices A and B , where

$$A = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -13 & 12 & 2 & 8 \\ -40 & 34 & 4 & 22 \\ 29 & -23 & 1 & -17 \\ 25 & -20 & -2 & -12 \end{pmatrix}.$$

i. Determine the trace of $A^T(B + 3A)$. [1]

ii. Determine the eigenvalues and corresponding eigenvectors of the matrix A . [2]

iii. Find the inverse of B using the Cayley–Hamilton theorem. [2]

4. (a) If $\vec{u} = (-2, 3, 6)$ and $\vec{v} = (3, -9, 4)$, find projection of $\vec{u}$ on $\vec{v}$ and $\vec{v}$ on $\vec{u}$. [5]

(b) Consider an equation of a circle, $x^2 + y^2 = 6^2$ and the region $-6 \leq x, y \leq 6$.

i. Find the total number of integer points on, in and outside the circle. [2]

ii. Print all the points on, in and outside the circle. [2]

iii. Sketch the circle and all the points in one set of axes. [2]

(c) Consider a second degree equation: [4]

$$f(x, y) \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

(i) Input the values of a, h, b, c, g and f and then test whether the second degree equation represents a pair of straight line or not.

(ii) If the equation in (i) represents a pair of straight line, then determine the equation of these two lines.