

Kishoreganj University

Department of Mathematics

3rd Year 1st Semester B.Sc. (Hons) Final Examination 2025

Course Code: MAT 3101 Course Title: Functional Analysis

Total Marks: 70 Time: 03.00 Hours Credits: 3.00

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Define a linear operator. Prove that the set of all linear operators on a finite-dimensional vector space forms a vector space. [7]
(b) Define a basis and its dual on a finite dimensional linear space. Find the dual basis of the vectors $x_1 = (1, -1, 3)$, $x_2 = (0, 1, -1)$ and $x_3 = (0, 3, -2)$ in $\mathbb{R}^3$. [7]
2. (a) Define an inner product and hence an inner product space, respectively. Let X be the set of all complex-valued continuous functions $x = x(t)$ on $[a, b]$. Show that X becomes an inner product space with inner product defined by $\langle x, y \rangle = \int_a^b x(t) \overline{y(t)} dt$. [5]
(b) If X is a complex inner product space, derive the polarization identity. [2]
(c) Define Hilbert space. Show that the polarization identity in (b) satisfies the properties of a Hilbert space. [7]
3. (a) State and prove the Banach Fixed Point Theorem. [8]
(b) Deduce an application of the theory described in (a). [6]
4. (a) Define sequence space l^p . Show that l^2 is a Hilbert space. [6]
(b) Define Cauchy sequence. When a Cauchy sequence which has a convergent subsequence is itself convergent? [4]
(c) Prove that, $C[a, b]$ is a complete metric space. [4]
5. (a) State and prove the closed graph theorem. [6]
(b) Exhibit an example which shows that a closed linear operator need not be continuous. [4]
(c) State and prove Riesz's lemma. [4]
6. (a) Let X and Y be two Banach spaces and $T : X \rightarrow Y$ a continuous linear operator of X onto Y . Prove that T is an open mapping. [7]
(b) State and prove the uniform boundedness principle. [7]
7. (a) Show that completeness is preserved under isometric isomorphism. [7]
(b) Suppose, $1 < p < \infty$ and a sequence $\{\alpha_k\}$ of scalars is given such that $\sum_1^\infty \alpha_k \xi_k$ is convergent whenever $x = \{\xi_k\}$ is an element of the space l^p . Then using closed graph theorem to prove that $y = \{\alpha_k\}$ is an element of l^q such that $\frac{1}{p} + \frac{1}{q} = 1$. [7]
8. (a) State and prove the Baire Category theorem. [6]
(b) State and prove the Riesz representation theorem. [8]

Kishoreganj University

Department of Mathematics

3rd Year 1st Semester B.Sc. (Hons) Final Examination 2025

Course Code: MAT 3103 Course Title: Complex Analysis

Total Marks: 70 Time: 03.00 Hours Credits: 3.00

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) What is entire z -plane? Explain stereographic projection. [4]
- (b) Find all values of z for which $z^5 = -32$, and also locate these values in the complex plane. [5]
- (c) Suppose we choose the principal branch of $\sin^{-1} z$ to be that one for which $\sin^{-1} 0 = 0$. Prove that [5]

$$\sin^{-1} z = \frac{1}{i} \ln \left(iz + \sqrt{1 - z^2} \right).$$

2. (a) Define single and multiple valued complex function with examples. [3]
 - (b) Prove the necessary and sufficient condition for a complex function $f(z) = u(x, y) + iv(x, y)$ being analytic in a region. [5]
 - (c) Suppose c_1 and c_2 are any real constants. Determine the set of all points in the z plane that map into the lines (i) $u = c_1$, (ii) $v = c_2$ in the w plane by means of the mapping function $w = z^2$. Illustrate by considering the cases $c_1 = 4, -2$ and $c_2 = 4, -2$. [6]
3. (a) State and prove the Cauchy-Goursat theorem for any simple closed curve. [6]
 - (b) Define winding number. Evaluate $\oint_C \frac{dz}{(z-2)^4}$ where C is the circle having center at origin and radius 1. [4]
 - (c) State the Rouché's theorem. Using Rouché's theorem, determine the number of zeros of the polynomial $z^6 + 5z^4 + z^3 - 2z$ inside the circle $|z| = 1$. [4]
4. (a) Let $f(z)$ be analytic inside and on the boundary C of a simply-connected region $\mathcal{R}$. Prove that [6]

$$f^{(n)}(a) = \frac{n!}{2\pi i} \oint_C \frac{f(z)}{(z-a)^{n+1}} dz \quad n = 0, 1, 2, 3, \dots$$

- (b) Evaluate $\oint_C \bar{z}^2 dz$ around the circles (i) $|z| = 1$, (ii) $|z - 1| = 1$. [5]
 - (c) Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 3$. [3]
5. (a) Find a Taylor-series expansion of the function $f(z) = \frac{1}{3z}$ about the point $z = 4i$, and calculate the radius of convergence. [7]
 - (b) Find the Laurent series of the function $f(z) = \frac{z+4}{z^2(z^2+3z+2)}$ in $1 < |z| < 2$. [7]
Classify the singularity at $z = 0$ and calculate the residue.

6. (a) State and prove Cauchy Residue theorem. [5]
- (b) Evaluate $\int_0^{2\pi} \frac{1}{5+3\sin\theta} d\theta$ using Contour and Residue. [5]
- (c) Evaluate $\oint_C \frac{e^z}{(z^2+\pi^2)^2} dz$ where C is the circle $|z| = 4$. [4]
7. (a) Define Harmonic function and conjugate harmonic function. Find a harmonic conjugate $v(x, y)$ of the harmonic function $u(x, y) = x^3 - 3xy^2$. Write the resulting analytic function in terms of z . [6]
- (b) If $f(z) = u(x, y) + iv(x, y)$ is analytic on a region A then show that both u and v are harmonic functions on A . [4]
- (c) Show that an analytic function with constant modulus is constant. [4]
8. (a) Define singularities. Describe different kind of singularities with example. [3]
- (b) Prove that $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$. [5]
- (c) Let the rectangular region $\mathcal{R}$ in the z plane be bounded by $x = 0, y = 0, x = 2, y = 1$. Determine the region $\mathcal{R}'$ of the w plane into which $\mathcal{R}$ is mapped under the transformations:
 (i) $w = z + (1 - 2i)$, (ii) $w = \sqrt{2}e^{\pi i/4}z$, (iii) $w = \sqrt{2}e^{\pi i/4}z + (1 - 2i)$. [6]

Kishoreganj University
Department of Mathematics

3rd Year 1st Semester B.Sc. (Hons) Final Examination 2025

Course Code: MAT 3105 Course Title: Ordinary Differential Equation II

Total Marks: 70 Time: 03.00 Hours Credits: 3.00

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Consider the initial-value problem (IVP):

$$\frac{dy}{dx} = x(y-1)^{1/3}, \quad y(x_0) = y_0$$

- i. State the basic existence and uniqueness theorem of solutions for first-order ODEs. [2]
- ii. Determine the region in the (x, y) -plane where the theorem guarantees the existence of a unique solution. [3]
- iii. Show that if $y_0 = 1$, the IVP does not have a unique solution. Provide at least two distinct solutions for the case $y(0) = 1$. [3]

- (b) What is a direction field? Sketch the direction field corresponding to the differential equation $\frac{dy}{dx} = xy$. [6]

2. (a) Define an ordinary point and a singular point of a linear differential equation. Locate and classify all the singular points of the equation [7]

$$x^2(x-2)^2y'' + 2(x-2)y' + (x+1)y = 0.$$

- (b) Solve in series the differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$ by the method of Frobenius. [7]

3. (a) Express the polynomials $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. [4]

- (b) State and prove the generating function Legendre polynomials. [5]

- (c) Show that $\int_{-1}^1 P_m(x)P_n(x)dx = \begin{cases} \frac{2}{2n+1}, & \text{when } m = n \\ 0, & \text{when } m \neq n \end{cases}$ [5]

4. (a) Verify directly that representation $J_0(x) = \frac{1}{\pi} \int_0^\pi \cos(x \sin \theta) d\theta$ satisfies Bessel's equation of order $n = 0$. [5]

- (b) Show that $y = x^{-n/2} J_n(2\sqrt{x})$ satisfies the differential equation [5]

$$x\frac{d^2y}{dx^2} + (n+1)\frac{dy}{dx} + y = 0.$$

- (c) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$. [4]

5. (a) Prove that Hermite polynomials are orthogonal over $(-\infty, \infty)$ with respect to the weight function e^{-x^2} . [6]

- (b) For Hermite polynomial $H_n(x)$, show that [4]

$$(i) H_{2n+1}(0) = 0, \quad (ii) H_{2n}(0) = (-1)^n \frac{\Gamma(2n+1)}{\Gamma(n+1)}.$$

- (c) For Laguerre polynomial $L_n(x)$, prove that $L_n(x) = \frac{e^x}{n!} \cdot \frac{d^n}{dx^n}(x^n e^{-x})$. [4]

6. (a) Write down the Laguerre polynomial of order n . Deduce that the function $(1-t)^{-1} \exp\left[\frac{-tx}{1-t}\right]$ generates the Laguerre polynomials. [5]

- (b) Prove that, $L_n(x) = \sum_{r=0}^{\infty} (-1)^r \frac{n!}{(n-r)! (r!)^2} x^r$ is a solution of the following differential equation: $xy' + (1-x)y' + ny = 0$. [4]

- (c) Derive the recurrence relations: [5]

$$(i) (n+1)L_n(x) = (2n+1-x)L_n(x) - nL_{n-1}$$

$$(ii) L'_n(x) = -\sum_{r=0}^{n-1} L_r(x)$$

7. (a) Prove that [6]

$$(i) \int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2},$$

$$(ii) \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$

- (b) Define error function. Prove that in the case of error function [2]

$$\operatorname{erf}(-x) = -\operatorname{erf}(x)$$

- (c) Show that the integral representation of hypergeometric function is [6]

$$F(\alpha; \beta; \gamma; x) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)} \int_0^1 t^{\beta-1} (1-t)^{\gamma-\beta-1} (1-xt)^{-\alpha} dt$$

where $\gamma > \beta > 0$.

8. (a) Convert the following BVP into a system and write it in matrix form: [3]

$$y''' + 5y' - 3y = \cos x, \quad y(0) = -1, \quad y'(0) = 2, \quad y''(0) = 1$$

- (b) Solve the following IVP: $\frac{dx}{dt} = \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$. [5]

- (c) Find the general solution to the following non-homogeneous system: [6]

$$\frac{dx}{dt} = \begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix} x + t \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

Kishoreganj University

Department of Mathematics

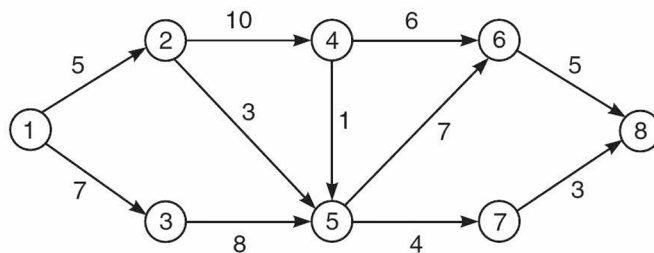
3rd Year 1st Semester B.Sc. (Hons) Final Examination 2025

Course Code: MAT 3107 Course Title: Linear Programming

Total Marks: 70 Time: 03.00 Hours Credits: 3.00

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Prove that the set $\{(x, y) : 2x^2 + y^2 \leq 12\}$ is a convex set. [4]
(b) Prove that the intersection of a finite collection of convex sets is convex. [4]
(c) What is feasible solution, basic feasible solution, and unbounded solution; explain with example? [6]
2. (a) What is convex combination? [3]
(b) Prove that a hyperplane is a convex set. [5]
(c) Let A be an $m \times n$ matrix and b a vector in R^m . Prove that the set of all solutions to the system of linear equations $Ax = b$ is a convex set. [6]
3. (a) Formulate the following LPP and convert it into standard form: A company has two grades of inspectors, I and II to undertake quality control inspection. At least 1,500 pieces must be inspected in an 8-hour day. Grade I inspector can check 20 pieces in an hour with an accuracy of 96%. Grade II inspector checks 14 pieces an hour with an accuracy of 92%. Wages of grade I inspector are 5\$ per hour while those of grade II inspector are 4\$ per hour. Any error made by an inspector cost 3\$ to the company. If there are, in all, 10 grade I inspectors and 15 grade II inspectors in the company, find the optimal assignment of inspectors that minimizes the daily inspection cost. [7]
(b) Given below is a network in which the figures written against the arrows indicate the cost of traveling in taka from the preceding to the following nodes. Formulate the linear programming model to find the least cost route from node 1 to node 8. [7]



4. (a) Use the simplex method to solve: [7]

$$\begin{aligned} \text{Minimize: } Z &= 3x_1 + x_2 + x_3 + x_4 \\ \text{Subject to: } -2x_1 + 2x_2 + x_3 &= 4 \\ 3x_1 + x_2 + x_4 &= 6 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Find an alternative optimal solution if one exists.

(b) Solve the following LP problems graphically: [7]

$$\begin{aligned} \text{Minimize: } Z &= 6x_1 + 5x_2 \\ \text{Subject to: } 3x_1 + x_2 &\leq 160 \\ x_1 &\leq 40 \\ x_2 &\leq 130 \\ x_1 &\geq 80 \\ x_1, x_2 &\geq 0 \end{aligned}$$

5. (a) Explain the Big M method and the Two-Phase method for finding an initial basic feasible solution. When is each method preferred? [6]

(b) Use the big M simplex method to solve: [8]

$$\begin{aligned} \text{Minimize: } Z &= 6x_1 + 3x_2 + 4x_3 \\ \text{Subject to: } x_1 &\geq 30 \\ x_2 &\leq 50 \\ x_3 &\geq 20 \\ x_1 + x_2 + x_3 &= 120 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

6. (a) Solve the following linear program by 2-phase simplex method (Upto Table 3): [8]

$$\begin{aligned} \text{Maximize: } Z &= 3x_1 + 4x_2 + 2x_3 \\ \text{Subject to: } x_1 + x_2 + x_3 + x_4 &\leq 30 \\ 3x_1 + 6x_2 + x_3 - 2x_4 &\leq 0 \\ x_2 &\geq 4 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

(b) State and prove the complementary slackness theorem. [6]

7. (a) Solve the following linear program by revised simplex method: [8]

$$\begin{aligned} \text{Maximize: } Z &= 5x_1 + 2x_2 + 3x_3 - x_4 + x_5 \\ \text{Subject to: } x_1 + 2x_2 + 2x_3 + x_4 &= 8 \\ 3x_1 + 4x_2 + x_3 + x_5 &= 7 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

(b) Write the dual of the following linear program [6]

$$\begin{aligned} \text{Maximize: } Z &= x_1 + 4x_2 + 3x_3 \\ \text{Subject to: } 2x_1 + 3x_2 - 5x_3 &\leq 2 \\ 3x_1 - x_2 + 6x_3 &\geq 1 \\ x_1 + x_2 + x_3 &= 4 \\ x_1 \geq 0, \quad x_2 \leq 0, \quad x_3 &\text{ unrestricted} \end{aligned}$$

8. (a) Solve by the dual simplex method: [8]

$$\begin{aligned}
 &\text{Minimize: } Z = x_1 + 2x_2 + 3x_3 + 4x_4 \\
 &\text{Subject to: } x_1 + 2x_2 + 2x_3 + 3x_4 \geq 30 \\
 &\quad \quad \quad 2x_1 + x_2 + 3x_3 + 2x_4 \geq 20 \\
 &\quad \quad \quad x_1, x_2, x_3, x_4 \geq 0
 \end{aligned}$$

- (b) Based on the following table discuss sensitivity by changing the objective function coefficient of a basic variable. [6]

c_B	Basis \ c_j	2	3	1	0	0	Constants
		x_1	x_2	x_3	x_4	x_5	
2	x_1	1	0	-1	4	-1	1
3	x_2	0	1	2	-1	1	2
$\bar{c}$ Row		0	0	-3	-5	-1	$Z = 8$

Kishoreganj University

Department of Mathematics

3rd Year 1st Semester B.Sc. (Hons) Final Examination 2025

Course Code: MAT 3109 Course Title: Mechanics

Total Marks: 70 Time: 03.00 Hours Credits: 3.00

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) Find the equation to the resultant of a system of forces in a plane. [7]
Given, three forces P, Q, R act along the sides of the triangle formed by the lines

$$x + y = 1, \quad y - x = 1, \quad \text{and} \quad y = 2.$$

Find the resultant and the equation of the line of action of the resultant.

- (b) A square of side $2a$ is placed with its plane vertical between two smooth pegs which [7]
are in the same horizontal line, at a distance c apart. Show that it will be in
equilibrium when the inclination of one of its edges to the horizon is either $\frac{\pi}{4}$ or,

$$\frac{1}{2} \sin^{-1} \left(\frac{a^2 - c^2}{c^2} \right).$$

2. (a) Determine the magnitude and direction of the resultant of two coplanar forces P [7]
and Q acting at an angle θ .

- (b) Three forces of magnitudes $10 N$, $15 N$, and $20 N$ act on a particle and keep it in [7]
equilibrium. Find the angle between the forces.

3. (a) Define catenary and common catenary. Obtain the intrinsic equation of a common [6]
catenary.

- (b) A uniform chain of length l is suspended from two points A and B in the same [3]
horizontal line so that either terminal tension is n times that at the lowest point.
Show that the span AB must be $\frac{l}{\sqrt{n^2-1}} \cdot \log \{n + \sqrt{n^2-1}\}$.

- (c) Find the Center of Gravity (C.G.) of the area included between the curve $y^2(2a - [5]
x) = x^3$ and its asymptotes.

4. (a) Find the C.G. of the area enclosed by the curves $y^2 = ax$ and $y^2 = 2ax - x^2$, lying [7]
in the first quadrant.

- (b) Find the center of mass of a hemisphere in which the density is proportional to the [7]
 n -th power of the distance from the center.

5. (a) A particle moves in a straight line OA with an acceleration which is always directed [7]
towards O and varies inversely as the square of its distance from O ; if initially the
particle were at rest at A , find its motion and the periodic time.

- (b) A particle of mass m is acted upon by a force $m\mu \left(x + \frac{a^4}{x^3} \right)$ towards the origin. If [7]
it starts from rest at a distance a , show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.

6. (a) Show that the (p, r) equation of the path is $f = \frac{h^2}{p^3} \frac{dp}{dr}$ using $\frac{d^2u}{d\theta^2} + u = \frac{f}{h^2u^2}$. [7]

- (b) A particle describes an ellipse under a force $\frac{\mu}{(\text{distance})^2}$ towards the focus. If it was projected with velocity V from a point distance r from the centre of force, show that its periodic time is $\frac{2\pi}{\sqrt{\mu}} \left(\frac{2}{r} - \frac{V^2}{\mu} \right)^{\frac{-3}{2}}$. [7]
7. (a) Prove that, the sum of the moments about O of the forces acting on the particle is equal to the rate of change of the moment of momentum of the particle about O . [7]
- (b) A block B of mass m can slide freely on a frictionless arm OA which rotates in a horizontal plane at a constant rate $\dot{\theta}_0$. Knowing that B is released at a distance r_0 from O , express as a function of r , (i) the component v_r of the velocity of B along OA , (ii) the magnitude of the horizontal force $\mathbf{F}$ exerted on B by the arm OA . [4]
- (c) Determine the equations of motion in terms of radial and transverse components. [3]
8. (a) Define central force and central orbit. A particle describes the curve $r^n = a^n \cos(n\theta)$ under a force P to the pole. Find the law of force. [7]
- (b) Define an apse, apsidal distance and apsidal angle on a central orbit. If a particle is projected from an apse at a distance a with the velocity zero from infinity under the action of central force μr^{-2n-3} , prove that the path is $r^n = a^n \cos(n\theta)$. [7]

Kishoreganj University

Department of Mathematics

3rd Year 1st Semester B.Sc. (Hons) Final Examination 2025

Course Code: MAT 3111 Course Title: Programming in C++

Total Marks: 70 Time: 03.00 Hours Credits: 3.00

Answer any five (05) questions. Numbers given in the right margin indicate the marks of the respective questions.

1. (a) What is Object-Oriented Programming (OOP)? State its advantages. [4]
(b) Differentiate between procedural and object-oriented programming. [5]
(c) What is debugging? Describe different types of errors in C++. [5]
2. (a) Write the complete classification of C++ data types in your own words as a nested list. [4]
(b) Define symbolic constant. Write down the rules to define a symbolic constant. Also, make your sense clear with the help of this constant. [6]
(c) Write a C++ program using variable as a constant. [4]
3. (a) Define increment and decrement operator. Make your sense clear with the use of those operators in a program. [6]
(b) Evaluate the following expressions step by step considering operator precedence and associativity in C++. [4]
(i) $d = 100 - (8 * 3 + 2) / (7 - 2) + (9 \% 4) * (2 + 1) - 6 / 2$
(ii) $e = (9 / 2 * 3 + 1) \% (4 + 2) - 5 * (3 - 1) / 2 + 7$
(c) Write a C++ program using nested if-else to find the largest of three numbers without using logical operators. [4]
4. (a) Why do we need decision making and looping? Write down the general form of for loop. Also, explain the additional features of for loop. [6]
(b) Write a C++ program using a do-while loop to repeatedly ask the user for a positive number until they enter one. [4]
(c) Write down the general form of Go to statement. Also mention a program using this statement. [4]
5. (a) What is call by value? Explain call by reference with example. Compare call by value and call by reference. [4]
(b) Explain pointer basics. How does pointer help in call by reference? [4]
(c) Assume that you want to generate a table of multiples of any given number. Write a program that allows to enter the number and then generates the table, formatting it into 10 columns and 20 lines. Interaction with the program should look like this (only the first three lines are shown): [6]

Enter a number: 7

7	14	21	28	35	42	49	56	63	70
77	84	91	98	105	112	119	126	133	140
147	154	161	168	175	182	189	196	203	210

6. (a) Define function. What are its advantages? Explain types of functions in C++. [5]
- (b) Differentiate between local and global variables. [4]
- (c) On a certain day the British pound was equivalent to \$1.487 U.S., the French franc was \$0.172, the German deutschemark was \$0.584, and the Japanese yen was \$0.00955. Write a program that allows the user to enter an amount in dollars, and then displays this value converted to these four other monetary units. [5]
7. (a) What are friend functions? List any three characteristics of a friend function. Write a C++ program where a friend function is used to add the private data members of two different classes Class A and Class B. [6]
- (b) Describe the mechanism of accessing data members and member functions in the following cases: [8]
- (i) Inside the main program.
- (ii) Inside a member function of the same class.
- (iii) Inside a member function of another class.
8. (a) Define a constructor. Write down the characteristics of constructors. [4]
- (b) What is inheritance? Explain the concept of reusability in the context of inheritance. Differentiate between single inheritance and multiple inheritance with suitable examples. [6]
- (c) Write a C++ program to demonstrate multiple inheritance. Create two base classes Marks (marks obtained) and Sports (sports score), and a derived class Result that inherits from both and displays the total. [4]